Deadline **Monday**, November 19.

APM 346 (2012) Home Assignment 7

This assignment is based on Lecture 22, Lecture 23 and Lecture 24 coming shortly.

Problem 1

(a) Find the solutions that depend only on r of the equation

$$\Delta u := u_{xx} + u_{yy} + u_{zz} = k^2 u,$$

where k is a positive constant. (*Hint:* Substitute $u = v/r$)

(b) Find the solutions that depend only on r of the equation

$$\Delta u := u_{xx} + u_{yy} + u_{zz} = -k^2 u,$$

where k is a positive constant. (*Hint:* Substitute $u = v/r$)

Problem 2

(a) Try to find the solutions that depend only on r of the equation

$$\Delta u := u_{xx} + u_{yy} = k^2 u,$$

where k is a positive constant. What ODE should satisftfy $u(r)$?

(b) Try to find the solutions that depend only on r of the equation

$$\Delta u := u_{xx} + u_{yy} = -k^2 u,$$

where k is a positive constant. What ODE should satisftfy $u(r)$?

Problem 3

(a) Solve

$$\Delta := u_{xx} + u_{yy} = 0 \qquad \text{in } r < a$$

$$u|_{r=a} = f(\theta).$$

where we use polar coordinates (r, θ) and $f(\theta) = \begin{cases} 1 & 0 < \theta < \pi \\ -1 & \pi < \theta < 2\pi. \end{cases}$

(b) Solve

$$\Delta := u_{xx} + u_{yy} = 0 \qquad \text{in } r > a$$

$$u|_{r=a} = f(\theta),$$

$$\max |u| < \infty.$$

where we use polar coordinates (r, θ) and $f(\theta) = \begin{cases} 1 & 0 < \theta < \pi \\ -1 & \pi < \theta < 2\pi. \end{cases}$

Problem 4

(a) Solve

$$\Delta := u_{xx} + u_{yy} = 0 \qquad \text{in } r < a$$

$$u_r|_{r=a} = f(\theta).$$

where we use polar coordinates (r, θ) and $f(\theta) = \begin{cases} 1 & 0 < \theta < \pi \\ -1 & \pi < \theta < 2\pi. \end{cases}$

(b) Solve

$$\Delta := u_{xx} + u_{yy} = 0 \qquad \text{in } r > a$$

$$u_r|_{r=a} = f(\theta),$$

$$\max |u| < \infty.$$

where we use polar coordinates (r, θ) and $f(\theta) = \begin{cases} 1 & 0 < \theta < \pi \\ -1 & \pi < \theta < 2\pi. \end{cases}$