

Math 344 Winter 2002

Problem Set 1

§2.6: 8, 15, 23, 29 (i.e. 6 divides $2^n + 3^n - 5^n$).

§2.7: 35, 38, 44

Chapt. 2 supplement (p. 103): 77

§7.1: 12, 25 (give a combinatorial argument as well), 27, 32, 33

§2.1 7, 11, 12, 26, 27, 32, 36, 37

A. Prove that

$$C(n+k, k) = \sum_{i=0}^k C(n, k-i)C(k, i)$$

in two ways:

- (i) algebraically by looking at the coefficient of $x^k y^n$ in the expansion of $(x+y)^{n+k} = (x+y)^n(x+y)^k$
- (ii) combinatorially by considering a set with n “blue” elements and k “red” elements and choosing k elements from it by first choosing blue ones and then red ones.

Deduce that

$$C(2n, n) = \sum_{i=0}^n C(n, i)^2$$