MAT 344

Challenge Set 1

The following questions illustrate some interesting mathematical counting ideas which will be the topic of future material in this course. Each question can be answered by simply writing down all of the possible cases step by step. Done correctly, this will lead you to the general principle involved. Try to prove your answer carefully.

1. Determine the number of positive integers less than or equal to 100 which are:
   - odd (that is, not divisible by 2)
   - not divisible by 2 or 5
   - not divisible by 2, 5, or 11

2. Find the number of ways of giving 4 different colour pens to 2 people a) without any restrictions b) so that each person gets at least one pen.

3. How many different addresses on Memory Lane can be constructed using the digits 0 through 9, assuming that no address can have more than 2 digits? (Note: Think carefully about what constitutes an address.)

4. Show that in any group of 13 people, at least two were born in the same month. (O.K., so if you think that's so easy, use the same principle to show that if 6 numbers are selected from the set of numbers {1,2,...,10} then one will divide another exactly!)

5. In a round-robin tournament every player plays every other player. Suppose we allow no ties, and we give each person one point for a win and 0 for a loss. If 4 people are in the tournament, find the sum of their scores after all the matches have been played. (Does this depend upon the outcome of the games?)

6. In how many ways can 5 men and 5 women be paired off at a dance into couples with each couple containing one member of each sex?

7. Suppose 6 people show up for a meeting. At the beginning of the meeting some of the people shake hands. Show that the number of people who shake hands with an odd number of the others is even.

8. What is the maximum number of pieces of pizza that a person can obtain by making 5 straight cuts with a pizza knife? (Note: the cuts need <u>not</u> pass through the centre of the pizza!! The pieces need not be the same size!!)

The following questions are intended to remind you about ideas with which you should be familiar already, and to get you writing some simple proofs. Try them, you'll like them.

9. For n a natural number, define $A_n=\{n, n+1,...,2n+1\}$. Find the union of all the A_n, for n from 0 to infinity. Similarly, find the intersection of the A_n.

10. Let X and Y be finite sets, and f a function from X to Y. Show that:
a) If f is 1:1, the card(X)≤card(Y)
b) If f is onto, the card(X)≥card(Y)
c) If f is 1:1 and onto, the card(X)=card(Y)
d) If card(X)=card(Y), the f is 1:1 if and only if f is onto.

11. Prove that for n sufficiently large, $1<\log_2 n < n < n\log_2 n < n^2 < n^3 < 2^n$.