Cosmic Censorship

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May 1, 2003

1 Introduction

One of the fundamental differences between the relativistic universe and the Newtonian one is the behaviour of information. In the Newtonian universe, the state at any point is affected by the state at all other points at the same time, so signals can be transmitted instantaneously.

Special relativity alone restricts the transmission of information, since it imposes a speed limit on any motion. Nonetheless, the light cone of a point expands to contain any point in space, so information about the original point is eventually accessible to any observer. However, general relativity allows for black hole regions—regions of finite size from which nothing can escape. This theory allows information which cannot be seen by many observers.

The cosmic censorship conjectures concern what is fit to be seen by observers. This property of general relativity allows singularities or other oddities to be hidden from observers. If true, the principle of cosmic censorship preserves the relevance of classical theory to the majority of the universe. It also has an appealing quantum-physics aesthetic to it, where, if the universe is not following the laws of physics or hasn't made up its mind what it's going to do, it only does so when it isn't going to get caught.

2 Singularities

A spacetime is said to have singularities if it possesses inextendible curves with finite affine length. They can be thought of as missing pieces of the spacetime. They disrupt the predictability of the spacetime, and allow uncontrollable information to enter into it.

Furthermore, many important singularities occur where the curvature blows up—that is, where the radius of curvature tends to zero. It is suggested that quantum effects become significant when the radius of curvature is on the order of the Planck length [?]. This introduces another sort of unpredictability beyond that in classical theory. In the region surrounding what classical theory would predict as a blow-up of curvature, classical theory itself ceases to be relevant, and a theory of quantum gravity is required to properly describe the behaviour of the universe.

For this reason, many physicists believe that singularities must be in some way hidden. Stephen Hawking asserts that

> God abhors a naked singularity. [**?**]

In fact, this is his formulation of cosmic censorship. The unpredictability created by the presence of singularities is unsettling to physicists. Also, as gravitational collapse occurs approximately once a year in our galaxy, if classical general relativity could not be applied accurately to a spacetime containing singularities, or if singularities allowed unpredictability to enter into the universe at large, classical general relativity would be an extremely incomplete theory [**?**], unable to explain the behaviour of the universe, even on a large scale. This issue led to the original formulation of the cosmic censorship conjecture, known now as weak cosmic censorship.

3 The Cosmic Censorship Conjectures

3.1 Weak Cosmic Censorship

In 1969, Penrose hypothesized that all singularities in physically realistic spacetimes are hidden inside black holes. In a spacetime $\mathcal{M}$, the black hole region is the part not in the causal past of future null infinity, $\mathcal{M} \setminus J^{-}\,[\mathcal{J}^{+}]$. This serves to restrict the area that the uncontrollable information from the singularity can reach. No signal from a singularity should be able to reach null infinity. Singularities should then have no effect on anything outside of their event horizons, and most of a classical general relativistic universe should be predictable.

This hypothesis is often formalized in terms of strong asymptotic predictability. A physically reasonable, generic set of initial conditions (the exact definitions of which are highly debatable) should produce a maximal Cauchy development that is asymptotically flat, and its unphysical spacetime should contain an open region which is globally hyperbolic [**?**]. Any open subset of the unphysical spacetime includes spacial infinity i_0. This can be interpreted to mean that such a spacetime contains a region that includes everything in the spacetime far enough out and contains no uncontrollable information, that is, no singularities.

Many physicists have become emotionally attached to this hypotheses; in fact, in 1991, Stephen Hawking entered into the following bet:

> Whereas Stephen W. Hawking firmly believes that naked singularities are an anathema and should be prohibited by the laws of classical physics,
>
> And whereas John Preskill and Kip Thorne regard naked singularities as quantum gravitational objects that might exist unclothed by horizons, for all the Universe to see,

> Therefore Hawking offers, and Preskill/Thorne accept, a wager with odds of 100 pounds sterling to 50 pounds sterling, that when any form of classical matter or field that is incapable of becoming singular in flat spacetime is coupled to general relativity via the classical Einstein equations, the result can never be a naked singularity.
>
> The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable concessionary message . . . [?]

Hawking lost the bet, due to the calculational work of Matthew Choptuik [?]. Choptuik showed that black holes scaled in mass—that it was possible to create a black hole of arbitrarily small mass, violating cosmic censorship [?]. However, this, and every other counterexample to weak cosmic censorship to date has been in some way contrived or unphysical. In this case, the spacetimes were constrained to be perfectly spherically symmetric. Such solutions are a set of measure zero. Results of Penrose and Simpson on the Kerr spacetime have suggested that the existence of naked singularities is unstable under asymmetric perturbation [?]. Others are dismissed as unphysical because they lead to singularities even in the special relativistic universe [?]. As singularities should never occur in special relativity, any system that exhibits them cannot be a physically valid one.

Despite God or Stephen Hawking's abhorrence of naked singularities, Penrose is more optimistic about the possibility. He argues that if naked singularities exist, their properties could help us understand the big bang singularity. Others [?] argue that, whether the cosmic censorship conjectures are true or not, they allow us to 'sweep under the rug' anomalies which classical theory cannot deal with. Eardley compares them to renormalization in quantum physics, and suggests that they may facilitate further progress in classical theory before quantum gravity can explain the behaviour of singularities [?].

3.2 Strong Cosmic Censorship

The condition that any singularity be hidden inside a black hole puts limits on the unpredictable region of a spacetime, but it is insufficient to rule out unpredictability in classical theory. As a black hole is simply a region that is not visible from infinity, the assertion that any singularity is hidden inside a black hole merely asserts that any observer who has observed a singularity is destined to fall into it. However, this could happen over cosmological timescales. For classical general relativity to be a complete theory, an observer who has fallen into a black hole should still be provided with a theory to explain what is observed. For this reason, Penrose formulated a stronger version of cosmic censorship [?], asserting that no singularity should ever be visible to any observer.

Strong cosmic censorship has been formulated in several different ways. The mathematical formulation of the physical idea appears to be important to its proof or disproof [?].

3.2.1 Wald's Definition of Strong Cosmic Censorship [?]

Wald [?] states strong cosmic censorship physically as

> All physically reasonable spacetimes are globally hyperbolic, i.e., apart from a possible initial singularity ... no singularity is ever "visible" to any observer. [?]

Wald formalizes this in terms of extendibility of the maximal Cauchy development. If a spacetime can be extended beyond its maximal Cauchy development, then the future is unpredictable, which is exactly what strong cosmic censorship seeks to rule out. However, the Taub and Kerr universes are both extendible in this way. In the extension of the Taub universe, however, strong causality is violated, which is a sign that this extension might not be a physically meaningful one. In the extension of the Kerr universe, an observer on the Cauchy horizon can receive a signal from anywhere in the initial data set. This suggests one can make a small perturbation in the original data set could create a singularity over the Cauchy horizon, destroying the extendibility; in fact, this has been shown in the Reissner-Nordström universe. As the extendibility would then not be stable under small perturbations, it is not generic. Wald's formalization is that, given a physically reasonable initial data set Σ (for definitions of physical reasonability which are given),

> ...if the maximal Cauchy development of this initial data set is extendible, for each $p \in H^+(\Sigma)$ in any extension, either strong causality is violated at p or $\overline{I^-(p) \cap \Sigma}$ is noncompact.

3.2.2 Penrose's Definition of Strong Cosmic Censorship

Penrose formulates strong cosmic censorship in terms of indecomposable pasts (**IP**s) and indecomposable futures (**IF**s) [?]. Penrose's approach differs significantly from Wald's in that he treats the entireity of spacetime as a geometric object, where Wald begins with initial data and considers properties of its

A past set S is a set such that $I^-[S] = S$. An indecomposable past is such a set which cannot be expressed as the union of two past sets not equal to S. If there is a point $p \in \mathcal{M}$ such that $S = I^-(p)$, then S is a proper **IP**, or a **PIP**. Otherwise, S is a terminal **IP**, or a **TIP**. Indecomposable futures, or **IF**s, **PIF**s and **TIF**s are defined equivalently.

If $\mathcal{M}$ is strongly causal (usually considered a reasonable physical assumption), it is past distinguishing and future distinguishing—that is, both **PIP**s and **PIF**s correspond one-to-one with points in $\mathcal{M}$. We can then identify points in $\mathcal{M}$ with **PIP**s and with **PIF**s. **TIP**s and **TIF**s can be thought of as ideal points representing the future causal boundary $\partial^+\mathcal{M}$ and the past causal boundary $\partial^-\mathcal{M}$ respectively.

It can be shown that all **IP**s can be expressed as $I^{-}[\gamma]$ for some timelike curve γ. For a **PIP**, γ is a curve ending at the point the **PIP** is identified with. For a **TIP**, γ has no end point. This means that its limit is either a point at infinity or a singularity, so we can identify **TIP**s with such points. If γ has infinite affine length it is an ∞-**TIF**, and if it has finite length, it is a singular **TIF**. Again, ∞-**TIP**s and singular **TIP**s are defined similarly. A singular point will be identified with a **TIP** or a **TIF**.

To exclude the big bang from the set of naked singularities, we assert that any singularity we wish to exclude from our theory must have a point p to its past. For a singularity to be observable, there must be another point q with the singularity in its past. Thus it seems fair to define a naked singular **TIP** R as one with points p and q such that $p \in R$ and $R \subset I^{-}(q)$. Likewise, a naked singular **TIF** S is defined as one with $S \subset I^{+}(p)$ and $q \in S$.

As R is a past set, it must contain such a point p. As $I^{-}(q)$ is a **PIP**, a naked singular **TIP** can be defined as one contained in a **PIP**. Likewise, a naked singular **TIF** can be defined as one contained in a **PIF** [**?**].

We can define causal and chronological relation between **TIP**s by analogy with **PIP**s [**?**]. R causally precedes S if $S \subset R$ and R chronologically precedes S if $S \subset I^{+}(r)$ for some $r \in R$. Then $\partial^{+}\mathcal{M}$ is achronal if no two **TIP**s are chronologically related.

If a **TIP** R is chronologically preceded by another **TIP** then by definition it is contained in some **PIP** contained in that **TIP**. Conversely, if R is contained in some **PIP** $I^{-}(x)$, then if γ is a future-endless timelike curve extending from x, then $R \subset I^{-}(x)$ with $x \in I^{-}[\gamma]$. As $I^{-}[\gamma]$ is a **TIP**, R is chronologically preceded by a **TIP**. So $\partial^{+}\mathcal{M}$ is achronal, or nowhere timelike, if no **TIP** is contained in any **PIP**, which is the strong cosmic censorship condition on **TIP**s. The equivalent for **TIF**s is very similar.

This formulation can be understood intuitively. If the boundary is timelike at any point, there will be points in $\mathcal{M}$ to the past and to the future of that point.

Naked points at infinity can be defined similarly. Arguably, there is also reason to exclude these, as unpredictability from infinity is as distasteful to physicists as unpredictability from a singularity.

The exclusion of naked ∞-**TIP**s turns out to be equivalent to global hyperbolicity. If $\mathcal{M}$ has a **PIP** containing a **TIP**, then there is a future-inextendible timelike curve γ such that the **TIP** is $I^{-}[\gamma]$. Then $I^{-}[\gamma] \subset I^{-}(q)$ for some $q \in \mathcal{M}$. If p is a point on γ and $r_1, r_2, r_3, \ldots$ is a sequence of points proceeding indefinitely along γ. Because $r_i \in \gamma \subset I^{-}[\gamma] \subset I^{-}(q)$, r_i is in past of q, so there is a timelike curve from r_i to q. We can then define a causal curve ζ_i by joining this timelike curve to the segment of γ from p to r_i. If the ζ_is had a limit curve ζ, then if ζ' is the part of ζ in $I^{-}[\gamma]$, $I^{-}[\zeta'] = I^{-}[\gamma]$. As ζ' is a segment of ζ, which terminates at q, it cannot be future-inextendible. This contradicts that $I^{-}[\gamma]$ is a **TIP**. So there can be no such limit curve, and so the space of causal curves from p to q is not compact. As this is one of the consequences of global hyperbolicity, $\mathcal{M}$ is not globally hyperbolic.

Conversely, assume that $\mathcal{M}$ is not globally hyperbolic. Global hyperbolicity is equivalent to the condition that $I^+(p) \cap I^-(q)$ has compact closure. So $\mathcal{M}$ must have a p in past of a q such that $I^+(p) \cap I^-(q)$ does not have compact closure. It can then be shown that there is a p' in past of p such that $p' \notin D^-(\partial I^-(q))$. Thus there is a future-inextendible curve γ from $p\prime$ which does not cross $\partial I^-(q)$. As $p\prime$ is in past of p and hence q, γ must remain in past of q. So $I^-(p\prime) \subset I^-(q)$. This gives us a **TIP** contained within a **PIF**.

Likewise, the exclusion of naked ∞-**TIF**s is equivalent to global hyperbolicity. This means that both conditions excluding naked points at infinity are equivalent, so this condition has the appealing property of being time-symmetric. Global hyperbolicity, also equivalent to the existence of a Cauchy hypersurface, is closely related to the predictability of a space. These are all conditions of other formulations of cosmic censorship.

Penrose suggests a few more refinements. One is that no ∞-**TIP** contain a singular **TIP** (or the **IF** equivalent). This has the reassuring consequence that it forbids thunderbolts. A thunderbolt is a singularity which emits a pulse of infinite curvature travelling at the speed of light and destroying the universe as it goes. This singularity would be visible from $\mathcal{J}^+$, but would not violate previous definitions of strong cosmic censorship, so in fact the previous definitions of strong cosmic censorship are not strictly stronger than weak cosmic censorship. To make the definition more local, one could require also that no singular **TIP** contain another singular **TIP** (or the **IF** equivalent).

Penrose intentionally avoids stating the restrictions on the spacetime. The use of ideal points depends on $\mathcal{M}$ being future distinguishing and past distinguishing, so his formulation requires that spacetime satisfy this condition. This is a consequence of strong causality, however, which is generally considered a reasonable condition. He avoids refining the meaning of reasonable however, because he considers it unhelpful.

3.2.3 Dafermos' Work in Strong Cosmic Censorship

Dafermos [**?**] defines strong cosmic censorship as the property that any reasonable spacetime is predictable, giving two alternate definitions of predictability. The first is satisfied by his solution, but the second is not.

Dafermos defines a local future extension of a spacetime $(\mathcal{M}, g)$ as a spacetime $\left(\tilde{\mathcal{M}}, \tilde{g}\right)$ with $\left(\tilde{\mathcal{M}}, \tilde{g}\right) \supset (\mathcal{M}, g)$ and $I^-[\mathcal{M}'b] \setminus \mathcal{M}' \subset \mathcal{M}$ for $\mathcal{M}' = \tilde{\mathcal{M}} \setminus \mathcal{M}$.

The first definition of predictability of an initial value set Σ requires that there be no non-trivial local future extension $(\mathcal{M}, g)$ to the domain of dependence $D(\Sigma)$, where g has continuous curvature. The second requires that there be no non-trivial future extension $(\mathcal{M}, g)$ of its maximal Cauchy development $E(\Sigma)$, where g is continuous.

Dafermos' second formulation corresponds to Wald's. Like Wald's, both of his definitions are given in terms of evolution from initial data.

Dafermos examines the Reissner-Nordström spacetimes, which model a charged black hole, concentrating on the black hole region of the spacetime. (As this region is surrounded by an event horizon, his results do not apply to weak cosmic censorship.)

He found that the domain of dependence could not be extended. However, for some initial values, the maximal Cauchy development could. Dafermos invites the reader to choose his or her own interpretation of predictability, and come to his or her own conclusion about strong cosmic censorship.

References