

# APM426: GENERAL RELATIVITY

J. COLLIANDER

- (1) force-free particle motion/special relativity
  - (a) variational characterization of dynamics; Hamiltonian dynamics
    - (i) simplest case; Cartesian coordinates
    - (ii) general coordinates; Riemannian manifolds
  - (b) geodesics
    - (i) geodesic equation
    - (ii) geodesic deviation; curvature
  - (c) relativistic force-free particle motion
    - (i) spacetime structure; spacetime interval
    - (ii) simultaneity/colocation/Lorentz transformations
    - (iii) causal structure; light cone
    - (iv) proper time; proper length
    - (v) Lagrangian dynamics of relativistic free particle; 4-momentum.
  - (d) Newtonian gravity
    - (i) Newton's theory of gravity; violates speed of light upper bound
    - (ii) Kepler problem
- (2) manifolds, tensors
  - (a) manifolds, basic definitions
  - (b) tangent vectors; curves on a manifold; diffeomorphisms
  - (c) covectors/dual vectors/one-forms
  - (d) tensors
    - (i) operations on tensors: contraction, outer product
    - (ii) transformation properties
    - (iii) example: metric tensor
    - (iv) example: faraday tensor, electromagnetic field
    - (v) tensor notation
- (3) curvature
  - (a) covariant derivative
    - (i) partial derivative transforms badly
    - (ii) formal correction of partial derivative
    - (iii) algebraic definition of covariant derivative
    - (iv) many covariant derivatives exist on a manifold; connection
    - (v) parallel transport
    - (vi)  $\exists!$  metric compatible covariant derivative
  - (b) Riemann curvature tensor
    - (i) commutator of covariant derivatives  $\rightarrow$  curvature tensor
    - (ii) parallel transport around infinitesimal closed loop
    - (iii) connection determines curvature
    - (iv) algebraic symmetry properties of curvature tensor

- (v) counting active indices
- (vi) differential identity for curvature tensor; Bianchi identity
- (vii) contractions; Ricci tensor; Ricci scalar; Einstein tensor
- (c) example calculations:  $S^2$  in detail
  - (i) metric
  - (ii) metric compatible Christoffel coefficients; connection
  - (iii) connection determines curvature tensor
  - (iv) Ricci tensor; Ricci scalar
- (d) geodesics; force-free general relativistic particle motion (Stephani)
- (e) Riemannian normal coordinates at a point
- (4) general relativity
  - (a) cartoon overview of Lagrangian derivation of GR
  - (b) structural assumptions and fundamental objects
    - (i) spacetime: a Lorentz manifold  $(M, g)$
    - (ii) matter fields; local causality and energy positivity postulates
    - (iii) Lagrangian density
  - (c) integration on a manifold; metric and chart induced volume elements
  - (d) tensor density
  - (e) Examples of Lagrangian field theories
    - (i)  $\mathbb{R}$ -valued scalar field  $\rightarrow$  Klein-Gordon equation
    - (ii) no matter fields  $\rightarrow$  Vacuum Einstein equation
    - (iii) gravity + scalar field  $\rightarrow$  Einstein-Klein-Gordon system
    - (iv) electromagnetic field  $\rightarrow$  Maxwell's equations
    - (v) gravity + em field  $\rightarrow$  Einstein-Maxwell system
    - (vi) wave maps?
    - (vii)  $\mathbb{C}$ -valued scalar field; internal symmetry
    - (viii) em field +  $\mathbb{C}$ -valued scalar field  $\rightarrow$  Maxwell-Klein-Gordon system
    - (ix) Sketch: gravity +  $\mathbb{C}$ -valued scalar field + em field  $\rightarrow$  Einstein-Maxwell-Klein-Gordon system
    - (x) Sketch: Yang-Mills and other gauge fields
- (5) exact solutions
- (6) linearized field equations/gravitational waves
- (7) initial-value problem
- (8) cosmology
- (9) discussion of student papers