(1) special relativity
   (a) spacetime structure
      (i) light speed invariance; spacetime interval
      (ii) light cone
      (iii) Lorentz transformations; Poincaré group
      (iv) proper time/length
      (v) paradoxes and their resolution
   (b) mechanics
      (i) Lagrangian and Hamiltonian descriptions of classical mechanics
      (ii) relativistic mechanics; 4-momentum; relativistic free particle

(2) manifolds, tensors
   (a) manifolds, basic definitions
   (b) tangent vectors; curves on a manifold; diffeomorphisms
   (c) covectors/dual vectors/one-forms
   (d) tensors
      (i) operations on tensors: contraction, outer product
      (ii) transformation properties
      (iii) example: metric tensor
      (iv) example: faraday tensor, electromagnetic field
      (v) tensor notation

(3) curvature
   (a) covariant derivative
      (i) partial derivative transforms badly
      (ii) formal correction of partial derivative
      (iii) algebraic definition of covariant derivative
      (iv) many covariant derivatives exist on a manifold; connection
      (v) parallel transport
      (vi) \exists! metric compatible covariant derivative
   (b) Riemann curvature tensor
      (i) commutator of covariant derivatives → curvature tensor
      (ii) parallel transport around infinitesimal closed loop
      (iii) connection determines curvature
      (iv) algebraic symmetry properties of curvature tensor
      (v) counting active indices
      (vi) differential identity for curvature tensor; Bianchi identity
      (vii) contractions; Ricci tensor; Ricci scalar; Einstein tensor
   (c) example calculations: \( S^2 \) in detail
      (i) metric
      (ii) metric compatible Christoffel coefficients; connection
(iii) connection determines curvature tensor
(iv) Ricci tensor; Ricci scalar
(d) geodesics
(e) Riemannian normal coordinates at a point

(4) general relativity
(a) cartoon overview of Lagrangian derivation of GR
(b) structural assumptions and fundamental objects
   (i) spacetime: a Lorentz manifold \((M, g)\)
   (ii) matter fields; local causality and energy positivity postulates
   (iii) Lagrangian density
(c) integration on a manifold; metric and chart induced volume elements
(d) tensor density
(e) Examples of Lagrangian field theories
   (i) \(\mathbb{R}\)-valued scalar field \(\rightarrow\) Klein-Gordon equation
   (ii) no matter fields \(\rightarrow\) Vacuum Einstein equation
   (iii) gravity + scalar field \(\rightarrow\) Einstein-Klein-Gordon system
   (iv) electromagnetic field \(\rightarrow\) Maxwell’s equations
   (v) gravity + em field \(\rightarrow\) Einstein-Maxwell system
   (vi) \(\mathbb{C}\)-valued scalar field; internal symmetry
   (vii) em field + \(\mathbb{C}\)-valued scalar field \(\rightarrow\) Maxwell-Klein-Gordon system
   (viii) Sketch: gravity + \(\mathbb{C}\)-valued scalar field + em field \(\rightarrow\) Einstein-Maxwell-Klein-Gordon system
   (ix) Sketch: Yang-Mills and other gauge fields

(5) exact solutions
(6) discussion of student papers

University of Toronto