Decoupling theory is a recent development in Fourier analysis, created by Jean Bourgain and Ciprian Demeter. It fully answers a family of difficult questions whose resolution had seemed far in the future. The theory has several applications in analysis and number theory. As the starting point of the lectures, I’m going to take a conjecture about diophantine equations which was recently proven using decoupling. In the first lecture, I will explain how this number theory problem relates to an estimate in Fourier analysis, and try to give a sense of how decoupling attacks this estimate.

In the next two lectures, I will discuss the proof of decoupling. The proof of decoupling is based on some (fairly simple) geometric estimates that are applied at many different scales. The new and surprising thing in the proof is just how much leverage the argument gets by combining information from many different scales. Starting with examples, I will try to build up intuition for the proof, and especially for the role of multi-scale arguments.

The whole series will aim to be accessible to a general mathematical audience.