

DEPARTMENT OF MATHEMATICS
University of Toronto

Complex Analysis Exam

1.5 hours

September 6, 2016

There are three questions, all of equal value.
Show all your work.

1. Use contour integration to evaluate

$$\int_0^{\infty} \frac{\sin x}{x(x^2 + 4)} dx .$$

2. Determine how many roots the equation $e^z = 3z + 1$ has in $|z| < 1$ and prove your answer briefly.

3. Show that

$$f(z) = \sum_{n=1}^{\infty} \frac{z}{z^2 - n^2\pi^2}$$

defines an analytic function in $|z| < \pi$. Find the Taylor coefficients of f about $z = 0$, and show that radius of convergence of the Taylor series is π .