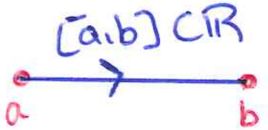
In this chapter, you met the following special cases of the general Stokes theorem: $\int_\Omega d\omega = \int_{\partial\Omega} \omega$

FTC:

"curve in ℝ"



$$\int_a^b F'(t)\,dt = F(b) - F(a)$$

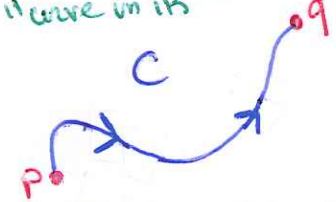
① $\int_a^b$ is the usual one-variable Riemann-Darboux integral

② $F : [a,b] \to \mathbb{R}$ C^1

③ $[a,b]$ is a segment line in $\mathbb{R}$

Gradient theorem

"curve in $\mathbb{R}^m$"



$$\int_C \vec{\nabla} f \cdot d\vec{x} = f(q) - f(p)$$

① $\int_C$ is the line integral for vector fields

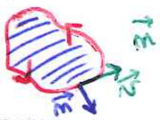
② C is an oriented curve in $\mathbb{R}^m$

③ $f : U \to \mathbb{R}$ C^1

$U \subset \mathbb{R}^m$ is an open subset containing C

- We want the surface to be on the left

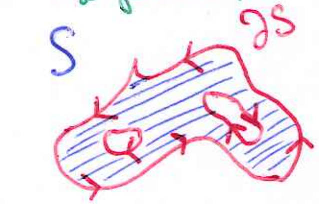
⟺ • If $\vec{v}$ is tangent compatible with the orientation, we want $\vec{n} = (v_2, -v_1)$ to point outward



$\vec{n}$ = rotation of $\vec{v}$ by $\frac{\pi}{2}$ clockwise

Green's theorem

"surface in $\mathbb{R}^2$"



$$\int_{\partial S} \vec{F} \cdot d\vec{x} = \iint_S \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}$$

or

$$\int_{\partial S} P\,dx + Q\,dy = \iint_S \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

① $S \subset \mathbb{R}^2$ is a planar regular region (surface in $\mathbb{R}^2$)

② ∂S is piecewise smooth and positively oriented

③ $\int_{\partial S}$ is the line integral for vector fields

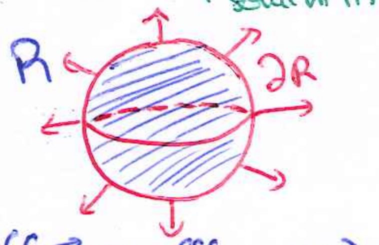
④ $\iint_S$ is the usual integral for 2-variable function $f : S \to \mathbb{R}$, $S \subset \mathbb{R}^2$

⑤ $\vec{F} : U \to \mathbb{R}^2$ C^1

$U \subset \mathbb{R}^2$ open with $S \subset U$

Divergence theorem

"solid in $\mathbb{R}^3$"



$$\iint_{\partial R} \vec{F} \cdot \vec{n} = \iiint_R \operatorname{div}(F)$$

① $R \subset \mathbb{R}^3$ is a regular region ("solid")

② ∂R is a piecewise smooth surface oriented by $\vec{n}$ the outward pointing normal unit vector.

③ $\iint_{\partial R}$ surface integral for vector fields

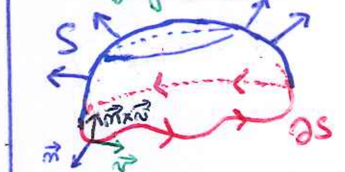
④ $\iiint_R$ usual integral for 3-variable functions $f : R \to \mathbb{R}$, $R \subset \mathbb{R}^3$

⑤ $\vec{F} : U \to \mathbb{R}^3$ C^1

$U \subset \mathbb{R}^3$ open, $R \subset U$

Stokes theorem

"surface in $\mathbb{R}^3$"



$$\int_{\partial S} \vec{F} \cdot d\vec{x} = \iint_S (\operatorname{curl} F) \cdot \vec{n}$$

① $S \subset \mathbb{R}^3$ is an oriented surface

② ∂S is the relative boundary of S with the positive orientation

It is an oriented curve in $\mathbb{R}^3$

③ $\int_{\partial S}$ is the line integral for vector fields

④ $\iint_S$ is the surface integral for vector fields

⑤ $F : U \to \mathbb{R}^3$ C^1

$U \subset \mathbb{R}^3$ open, $S \subset U$

We want $\vec{n} \times \vec{v}$ to point to the surface S