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INTRODUCTION TO TOPOLOGY:  
INTERIOR, CLOSURE AND BOUNDARY

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**Question 1:** Determine  $\overset{\circ}{S}$ ,  $\overline{S}$  and  $\partial S$  for the following sets?

1  $S = (0, 3] \subset \mathbb{R}$

2  $S = [0, 3) \subset \mathbb{R}$

3  $S = (0, 3) \subset \mathbb{R}$

4  $S = [0, 3] \subset \mathbb{R}$

5  $S = \{0, 3\} \subset \mathbb{R}$

6  $S = (\mathbb{R} \setminus \{0\}) \subset \mathbb{R}$

7  $S = (-\infty, 0) \subset \mathbb{R}$

8  $S = (-\infty, 0) \cup [3, +\infty) \subset \mathbb{R}$

9  $S = [0, 1) \cap \mathbb{Q} \subset \mathbb{R}$

10  $S = \left\{ \frac{1}{n} : n \in \mathbb{N}_{>0} \right\} \subset \mathbb{R}$

11  $S = [0, 3) \times \{0\} \subset \mathbb{R}^2$

12  $S =$   
 $\{(x, y) \in \mathbb{R}^2 : |y - x| > 1\}$   
 $\subset \mathbb{R}^2$

13  $S = B(\mathbf{0}, 1) \subset \mathbb{R}^n$

14  $S = \mathbb{R}^n \subset \mathbb{R}^n$

15  $S = \emptyset \subset \mathbb{R}^n$

**Question 2:** Let  $S = [0, 1) \cup (1, 2] \cup \{3\} \cup ([4, 5] \cap \mathbb{Q}) \subset \mathbb{R}$

Determine  $\overset{\circ}{S}$ ,  $\overset{\circ}{\overset{\circ}{S}}$ ,  $\overset{\circ}{\overline{S}}$ ,  $\overline{\overset{\circ}{S}}$ ,  $\overline{\overline{S}}$ ?

What do you notice? Is it possible to do *better*?