
PRELIMINARIES:
HOW TO VISUALIZE A MULTIVARIABLE FUNCTION



UNIVERSITY OF
TORONTO

September 12th, 2019

How to visualize a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$

- By its graph

$$\Gamma_f = \{(\mathbf{u}, v) \in \mathbb{R}^n \times \mathbb{R} : v = f(\mathbf{u})\} \subset \mathbb{R}^{n+1}$$

(efficient for $n \leq 2$.)

- By its level sets, for $c \in \mathbb{R}$,

$$L_c(f) = \{\mathbf{u} \in \mathbb{R}^n : f(\mathbf{u}) = c\} \subset \mathbb{R}^n$$

(efficient for $n \leq 3$.)

In the following slides, we'll see how the level sets $L_c(f) = \{(x, y) \in \mathbb{R}^2 : f(x, y) = c\}$ of a 2-variable function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ allow you to visualize its graph.

How to visualize a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$

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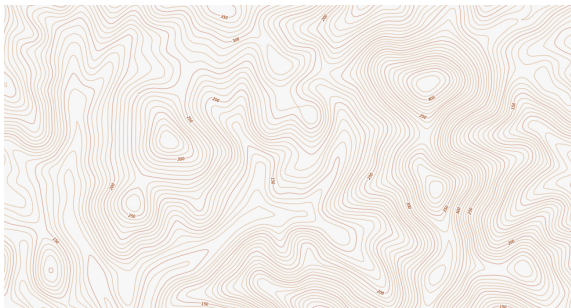
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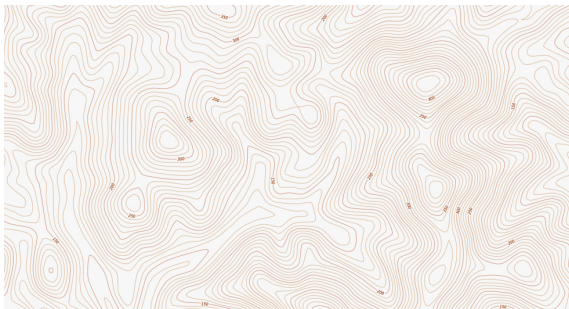
Let f be the function that associates to the location on earth at coordinates (x, y) its elevation $f(x, y)$.

We draw the level sets $f(x, y) = c$ for $c = 0, 10, 20, 30, 40, \mathbf{50}, 60, 70, 80, 90, \mathbf{100}, 110, \dots$

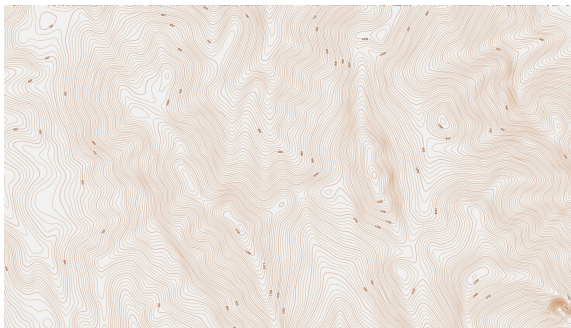
Graphs and level sets – 1



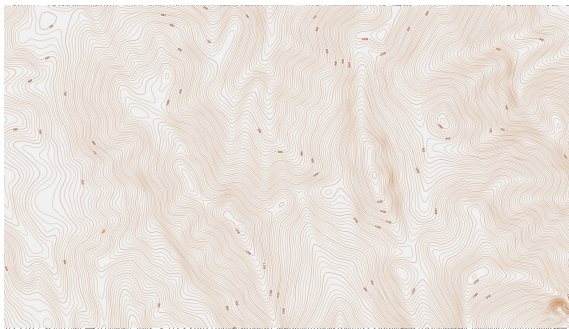
Graphs and level sets – 1



Graphs and level sets – 2



Graphs and level sets – 2



Graphs and level sets – 3

Let $f(x, y) = y^2 - x^2$

Draw some level sets $f(x, y) = c$ and then try to visualize the graph of f from them.

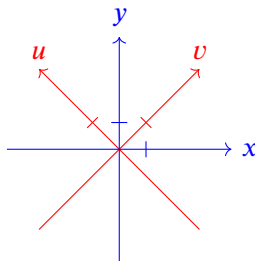
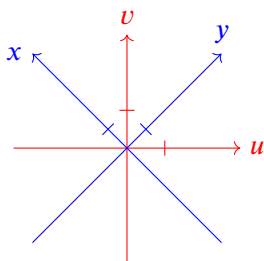
Graphs and level sets – 3

Let $f(x, y) = y^2 - x^2$

Draw some level sets $f(x, y) = c$ and then try to visualize the graph of f from them.

$$f(x, y) = y^2 - x^2 = (y - x)(y + x) = uv$$

So we just have to draw $uv = c$ (or $v = c/u$) and then to apply the change of variables (beware of the orientation).



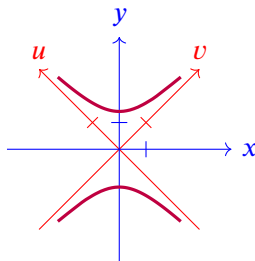
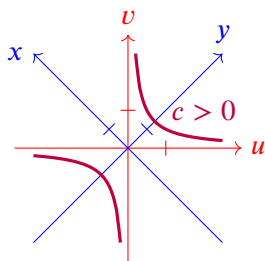
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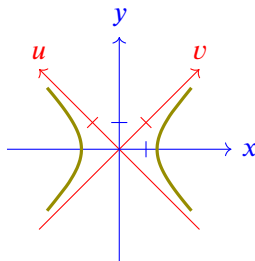
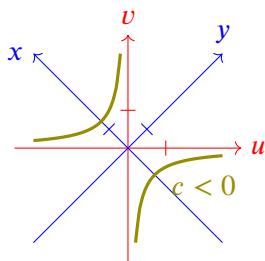
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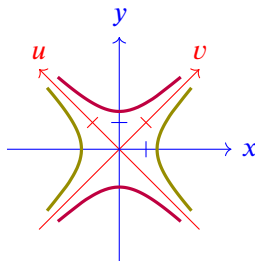
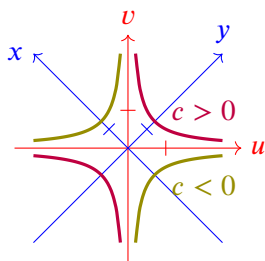
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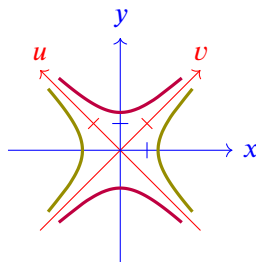
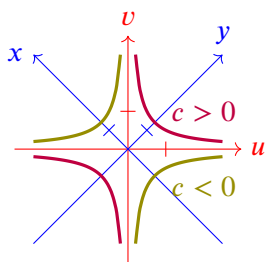
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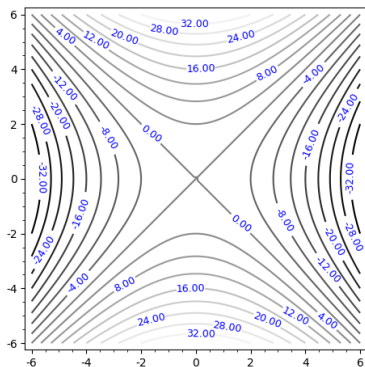


Or you directly recognize the equation of a hyperbola.

Graphs and level sets – 3

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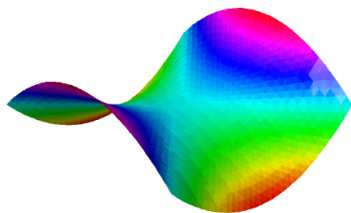
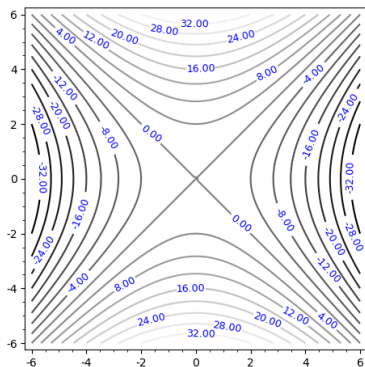
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- 1 Read the section 0.3 of the lecture notes.
- 2 Play with the interactive examples in the notes.
- 3 Work on the questions from the lecture notes (section 0.P).