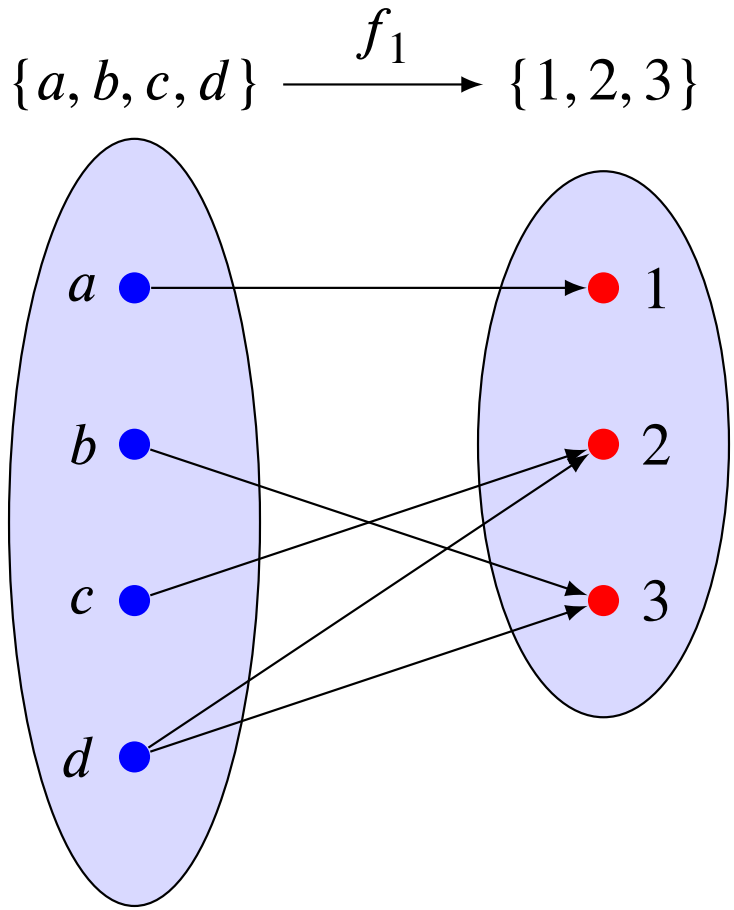
Preliminaries: Sets and functions

UNIVERSITY OF TORONTO

September 5th, 2019

Functions

Does it define a function?
If so, is it injective, surjective and/or bijective?
If it is bijective then give its inverse.



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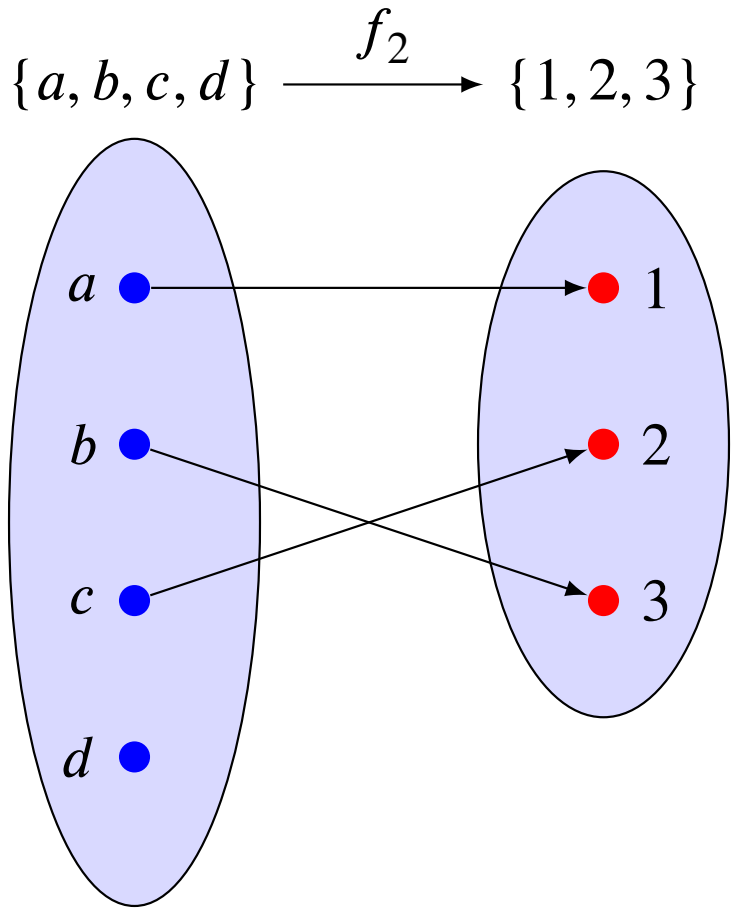

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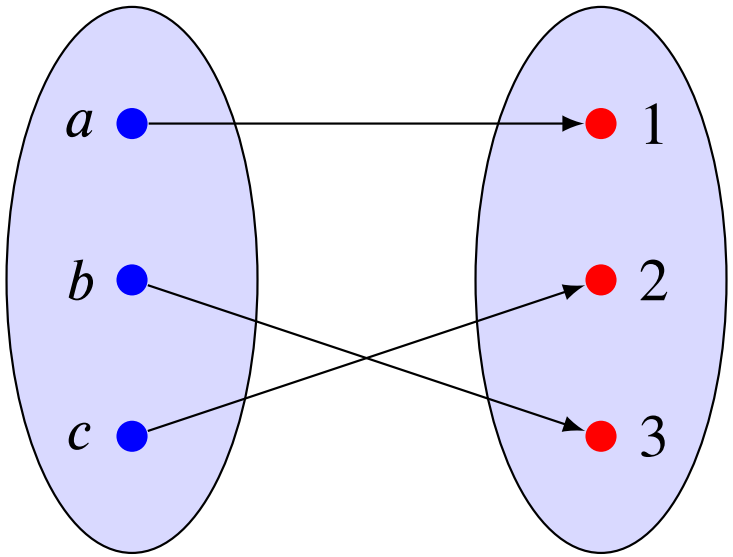
$$\{a, b, c\} \xrightarrow{f_3} \{1, 2, 3\}$$



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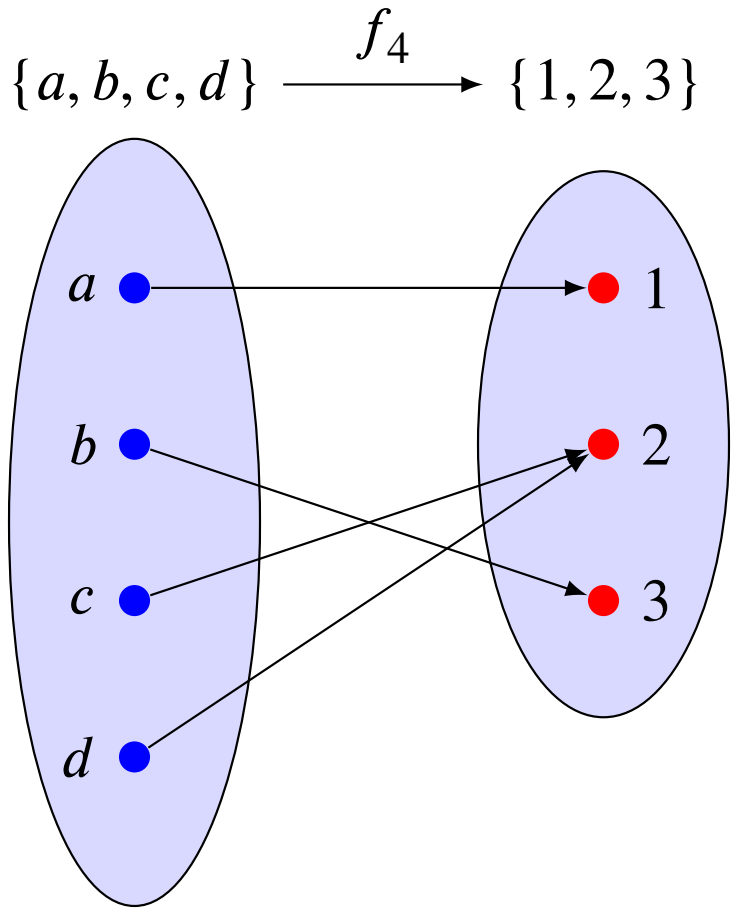

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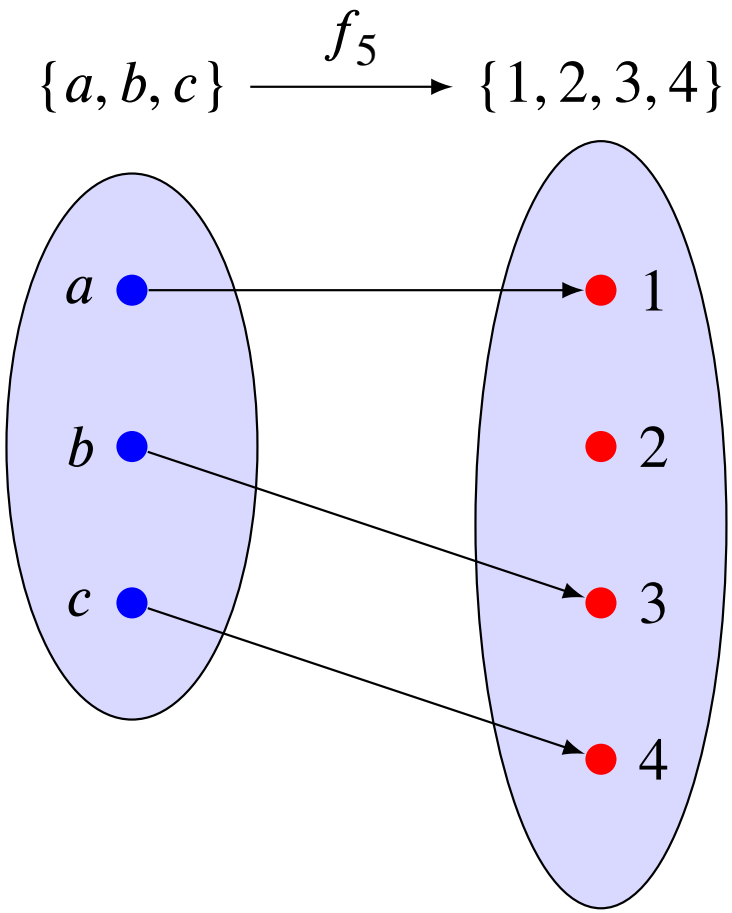

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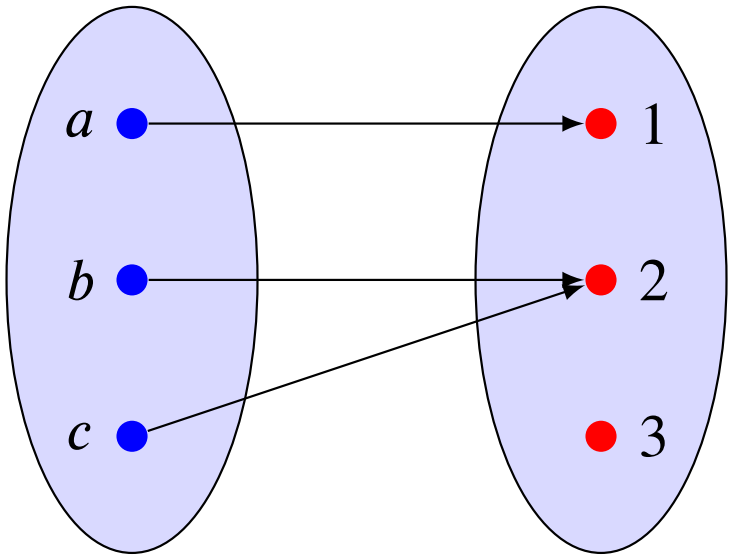
$$\{a, b, c\} \xrightarrow{f_6} \{1, 2, 3\}$$



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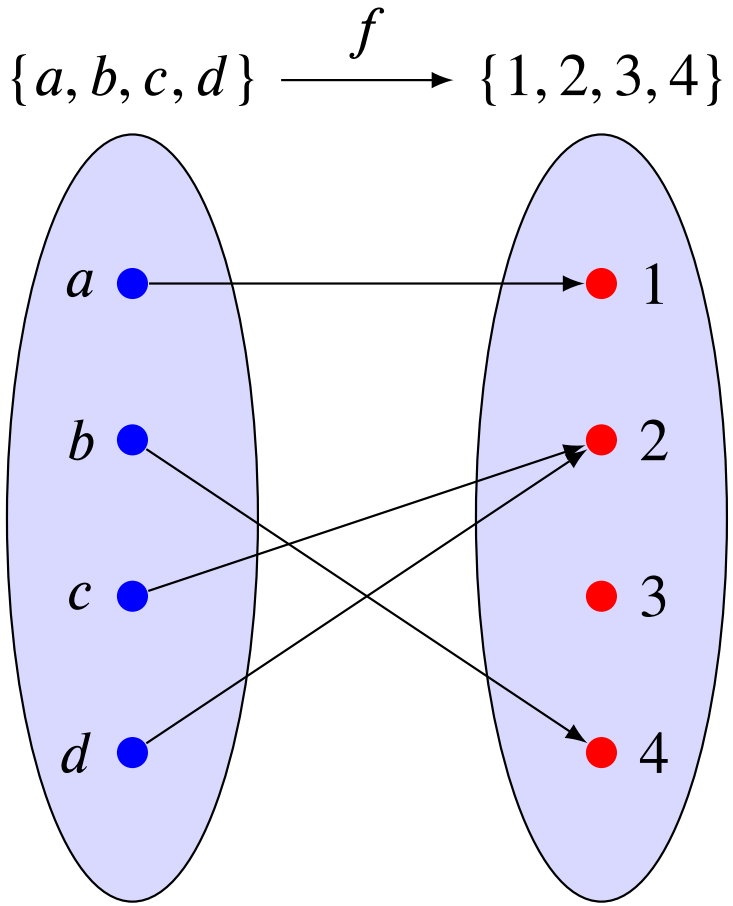
$$f_7 : \begin{array}{ccc} \mathbb{R}^2 & \to & \mathbb{R} \\ (x,y) & \mapsto & xe^y \end{array}$$

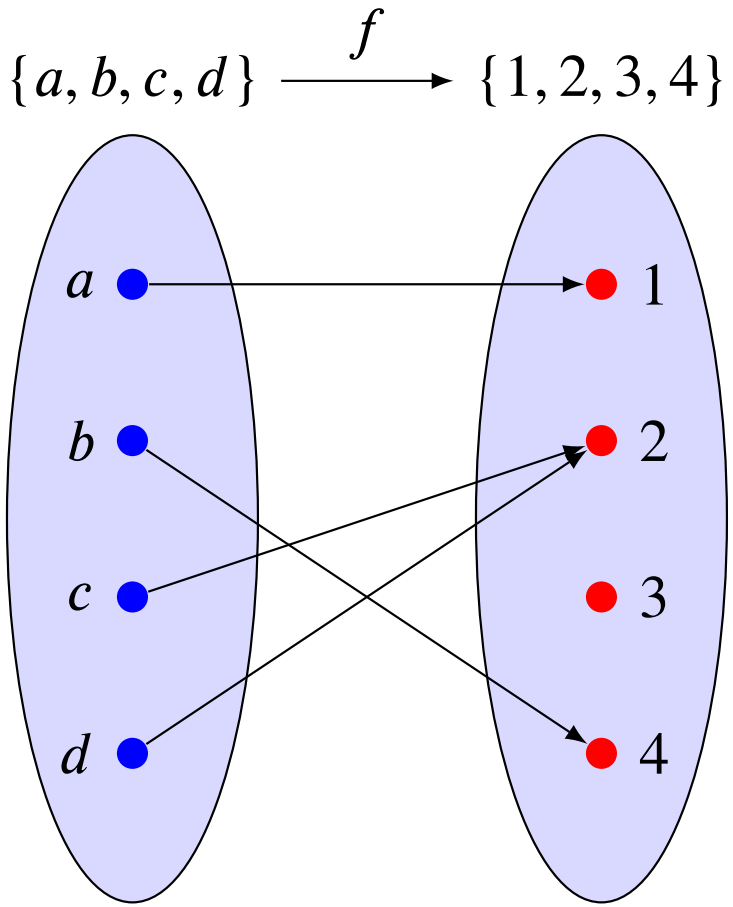
$$f_8 : \begin{array}{ccc} \mathbb{R}^2 & \to & \mathbb{R}^2 \\ (x,y) & \mapsto & \left(e^x, x^2\right) \end{array}$$

Image and inverse image of a set by a function



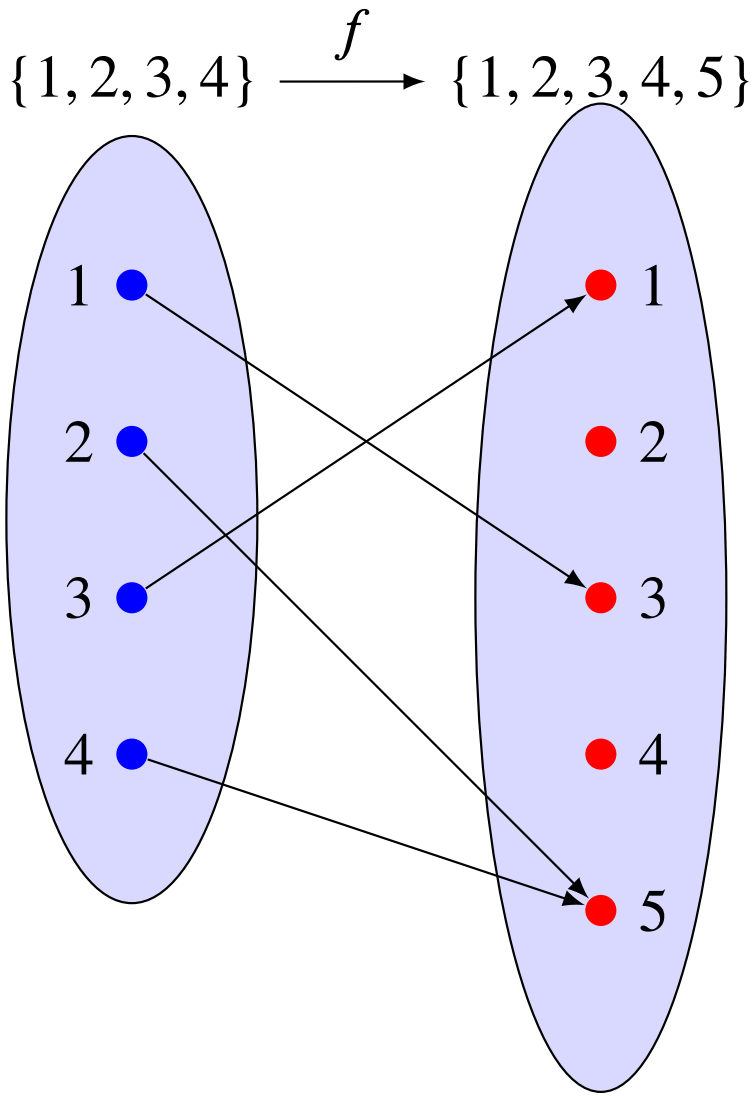
Compute $f(\{a, c, d\})$ and $f^{-1}(\{2, 3, 4\})$.

Image and inverse image of a set by a function



$f(\{a, c, d\}) = \{1, 2\}$ and $f^{-1}(\{2, 3, 4\}) = \{b, c, d\}$.

Compute the graph of the following function



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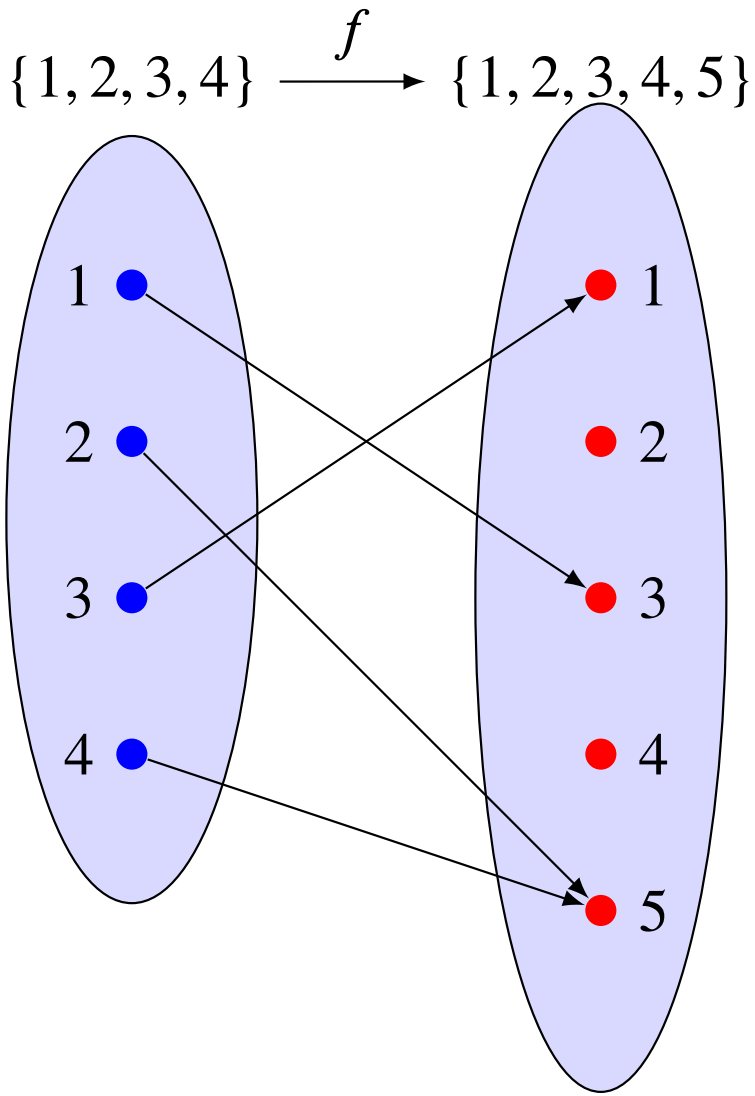
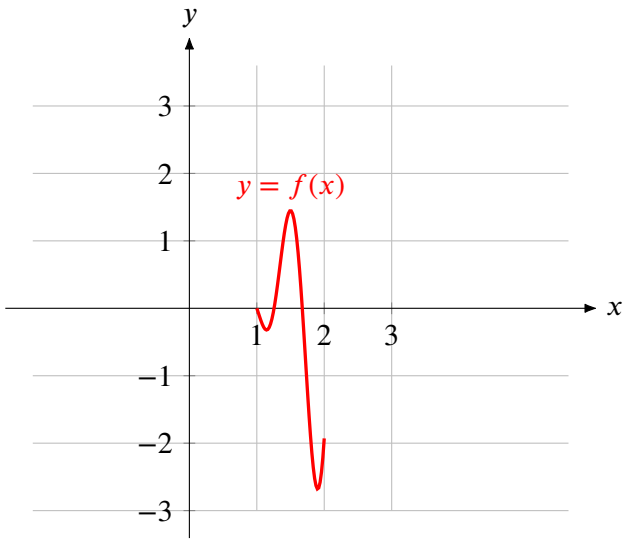
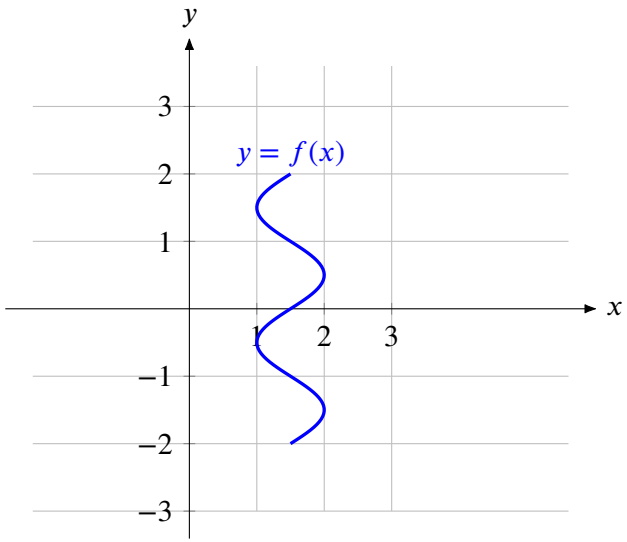


$$\Gamma_f = \{(1,3),(2,5),(3,1),(4,5)\}$$

Is it the graph of a function $f : [1, 2] \to \mathbb{R}$?

1. 

2. 

Injective/surjective/bijective functions

Are the following functions injective? surjective? bijective?
For a bijective one, give its inverse function.

1. $f: \begin{array}{ccc} \mathbb{R}^3 & \to & \mathbb{R}^2 \\ (x,y,z) & \to & (x,y) \end{array}$

2. $g: \begin{array}{ccc} \mathbb{R}^2 & \to & \mathbb{R}^2 \\ (x,y) & \to & \left(e^x, (x^2+1)y\right) \end{array}$

3. $h: \begin{array}{ccc} \mathbb{R}^2 & \to & \mathbb{R}^2 \\ (x,y) & \to & (x+y,-x) \end{array}$

4. $l: \begin{array}{ccc} \mathbb{N}_{\geq 0} & \to & \mathbb{Z} \\ n & \to & \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ -\frac{n+1}{2} & \text{otherwise} \end{cases} \end{array}$

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