University of Toronto

MAT188H1F TERM TEST

WEDNESDAY, NOVEMBER 16, 2005, 5:10 PM

Duration: 50 minutes

Aids Allowed: Casio 260, Sharp 520 or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [12 marks; 4 for each part] Find the following:

   (a) the angle between the vectors $\vec{u} = \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$

   (b) the area of the triangle with vertices

   $$P(1,1,1),\ Q(2,3,-2),\ R(0,2,2)$$

   (c) the shortest distance from the point $P(1,2,0)$ to the plane with equation $3x + y - 2z = 14$.

2. [10 marks] The parts of this question are unrelated.

   (a) [6 marks; 2 for each part] Write down the matrix for each of the following transformations:

   (i) a reflection in the line $y = -2x$

   (ii) a rotation through $\frac{\pi}{3}$

   (iii) a projection on the line $y = 3x$

   (b) [4 marks] Suppose $T : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ is a linear transformation such that

   $$T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \text{ and } T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

   Find $T^{-1}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

3. [13 marks] Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 4 & 1 & 0 \end{bmatrix}$.

   Find the eigenvalues and eigenvectors of A and use them to write down the general solutions of the following system of differential equations

   $$\begin{cases} f_1' = f_1 - f_2 + f_3 \\ f_2' = 2f_2 - f_3 \\ f_3' = 4f_1 + f_2 \end{cases}$$, where f_1, f_2 and f_3 are functions of x.

4.(a) [6 marks; 3 for each part] Decide if the following sets of vectors are linearly independent or dependent.

(i) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

(ii) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 1 \\ 5 \end{bmatrix} \right\}$

4.(b) [4 marks] Find a spanning set for the following subspace of $\mathbf{R}^4$:

$$U = \left\{ \begin{bmatrix} s+t \\ 2s-t \\ s \\ t \end{bmatrix} \middle| \, s \text{ and } t \text{ are real numbers} \right\}.$$

ANSWERS: 1(a) $\frac{\pi}{6}$ 1(b) $\frac{1}{2}\sqrt{38}$ 1(c) $\frac{9}{\sqrt{14}}$ 2(b) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

2(a)(i) $Q_{-2} = \frac{1}{5}\begin{bmatrix} -3 & -4 \\ -4 & 3 \end{bmatrix}$ 2(a)(ii) $R_{\pi/3} = \frac{1}{2}\begin{bmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$ 2(a)(iii) $P_3 = \frac{1}{10}\begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$

3. $\lambda_1 = 1; X_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}; \lambda_2 = -1; X_2 = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}; \lambda_3 = 3; X_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} = c_1X_1e^x + c_2X_2e^{-x} + c_3X_3e^{3x}$, where c_1, c_2 and c_3 are arbitrary constants.

4(a)(i) independent 4(a)(ii) dependent 4(b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

University of Toronto

MAT188H1F TERM TEST

WEDNESDAY, OCTOBER 12, 2005, 5:10 PM

Duration: 50 minutes

Aids Allowed: Casio 260, Sharp 520 or Texas Instrument 30 calculator.
Instructions: Fill in the information on this page, and make sure this test contains 5 pages. Present your **solutions** in the space provided. Use the backs of pages if you need more space. The value for each question is indicated in square brackets beside each question number.

TOTAL MARKS: 45

1. [10 marks] Consider the system of linear equations

$$\begin{array}{rcrcrcrcrcr} x_1 & + & x_2 & - & x_3 & + & x_4 & + & x_5 & = & 6 \\ & & x_2 & + & x_3 & & & - & x_5 & = & 2 \\ x_1 & + & 2x_2 & & & + & x_4 & & & = & 8 \\ 3x_1 & + & 2x_2 & - & 4x_3 & + & 3x_4 & + & 4x_5 & = & 16 \end{array}$$

   Find the reduced row-echelon form of the augmented matrix for this system, and then find the solution to the system.

2. [10 marks; 5 for each part] Find the following:

   (a) the inverse of $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$

   (b) $\det \begin{pmatrix} 3 & -1 & 0 & 0 \\ 1 & 1 & 7 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & 0 & 1 & 2 \end{pmatrix}$

3. [11 marks] The parts of this question are unrelated.

   (a) [5 marks] Use Cramer's Rule to find x_2 in the solution to the system of equations

$$\begin{array}{rcrcrcr} 2x_1 & + & x_2 & + & x_3 & = & 4 \\ -x_1 & + & x_2 & + & 4x_3 & = & -2 \\ x_1 & - & x_2 & + & 5x_3 & = & 2 \end{array}$$

   (b) [6 marks] Find all values of a for which the system of equations

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 1 \\ -x & + & ay & & & = & 3 \\ & & 6y & + & az & = & 8 \end{array}$$

   has infinitely many solutions.

4. [14 marks] Parts (a) and (b) of this question are unrelated.

(a) [4 marks] Suppose W, X, Y and Z are all $n \times n$ matrices. Find X in terms of W, Y and Z, if

$$2W + 3(X^T + Y)^{-1} = Z.$$

(b) [10 marks; 5 for each part] Suppose A and B are 5×5 matrices with $\det A = 2$ and $\det B = -1$. Find the following:

(i) $\det(A^2 B^3 A^T B^{-2})$

(ii) $\det(A^{-1} + \text{ adj } A)$

ANSWERS: 1. $\left(\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 2 & 4 \\ 0 & 1 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$; $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 4 + 2s - t - 2u \\ 2 - s + u \\ s \\ t \\ u \end{pmatrix}$, where s, t and u are parameters.

2.(a) $\begin{pmatrix} -1 & -3 & 2 \\ 2 & 3 & -2 \\ -1 & -1 & 1 \end{pmatrix}$ (b) -72 3.(a) $x_2 = 0$ (b) $a = 2$

4.(a) $X = (3(Z - 2W)^{-1} - Y)^T$ (b)(i) -8 (ii) $243/2$

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MAT188H1F Linear Algebra TERM TEST

Monday, November 22, 2004, 5:10 PM

1. [13 marks] Find the following:

   (a) [4 marks] the angle between the vectors $\vec{u} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

   (b) [5 marks] the area of the triangle with vertices

   $$P(1,1,1),\ Q(2,3,-5),\ R(0,2,2)$$

   (c) [4 marks] the projection of $\vec{v} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ on $\vec{d} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

2. [12 marks; 4 marks for each part] The parts for this quesiton are unrelated.

   (a) Find the (scalar) equation of the plane passing through the three points

   $$A(2,1,1),\ B(3,0,2),\ C(-1,2,4).$$

   (b) Find the vector equation of the line of intersection of the two planes with equations

   $$x + y + z = 6 \text{ and } 2x - y - 4z = 0.$$

   (c) What is the shortest distance from the point $P(1,2,0)$ to the line with vector equation

   $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = t \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix},$$

   where t is a parameter.

3. [13 marks] In this question all transformations are from $\mathbf{R^2}$ to $\mathbf{R^2}$. Find the following:

   (a) [3 marks] the standard matrix, A, of a reflection in the line with equation $y = x$.

   (b) [3 marks] the standard matrix, B, of a projection onto the line with equation $y = -x$.

   (c) [3 marks] $T(\vec{v})$, where T is the reflection of part (a) followed by the projection of part (b), and $\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

(d) [4 marks] $R^{90}(\vec{u})$, where $R = \begin{pmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{pmatrix}$ and $\vec{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

4. [12 marks; 4 marks for each part] The parts of this question are unrelated.

(a) Find a basis for null(A) if $A = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

(b) Find a basis of $\mathbf{R^3}$ which is contained in the spanning set

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 2 \end{pmatrix} \right\}$$

(c) Show that the set, U, of all vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ in $\mathbf{R^3}$ such that

$$\dim\left(\text{span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\}\right) = 2$$

is a subspace of $\mathbf{R^3}$, and find its dimension.

ANSWERS: 1(a) $\frac{5\pi}{6}$ (b) $\frac{7}{\sqrt{2}}$ (c) $\frac{7}{6}\vec{d}$

2(a) $2x + 3y + z = 8$ (b) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ (c) 2

3(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (b) $\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ (c) $\frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

4(a) $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$ (b) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

4(c) $U = \text{span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$ or $U = \text{null}\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$; $\dim(U) = 2$

University of Toronto

MAT188H1F TERM TEST

MONDAY, OCTOBER 25, 5:10 PM, 2004

Duration: 50 minutes

1. [12 marks; 4 for each part] Find the following:

   (a) the inverse of $A = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 2 & 1 \\ 0 & 2 & 3 \end{pmatrix}$

   (b) $\det \begin{pmatrix} 2 & 3 & 1 & 1 \\ 4 & 1 & -1 & 0 \\ -1 & 2 & 2 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$

   (c) an elementary matrix E such that $B = EA$ if

$$A = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 4 & 1 & 6 & 7 \\ 1 & -1 & 0 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & -11 & 10 & -1 \\ 1 & -1 & 0 & 2 \end{pmatrix}$$

2. [13 marks] Consider the system of linear equations $AX = B$, with

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

   (a) [4 marks] Find the reduced row-echelon form of the augmented matrix $(A|B)$ of this system.

   (b) [2 marks] What is the rank of the augmented matrix? _______

   (c) [1 mark] How many parameters are required to solve the system? _______

   (d) [4 marks] What is the solution to the system $AX = B$?

   (e) [2 marks] What are the basic solutions to the corresponding homogeneous system of equations $AX = O$?

3. [12 marks; 4 for each part] Find the following:

   (a) all values of a for which the homogeneous system of equations

$$\begin{array}{rcrcrcl} 2x_1 & & & + & x_3 & = & 0 \\ ax_1 & + & x_2 & & & = & 0 \\ 15x_1 & + & ax_2 & + & ax_3 & = & 0 \end{array}$$

   has non-trivial solutions.

(b) the characteristic polynomial of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(c) $\det(A^2B^{-3}A^TB^5)$, if A and B are $n \times n$ matrices with

$$\det A = -1 \text{ and } \det B = 2.$$

4. [13 marks] Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

Find a diagonal matrix D and an invertible matrix P such that $D = P^{-1}AP$.

ANSWERS: 1(a) $\begin{pmatrix} -2 & -3 & 3 \\ -3/2 & -3/2 & 2 \\ 1 & 1 & -1 \end{pmatrix}$ 1(b) 2 1(c) $\begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2(a) $\left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$ 2(b) 3 2(c) 2

2(d) $X = \begin{pmatrix} t \\ -s-t \\ s \\ 1-t \\ t \end{pmatrix}$, where s and t are parameters. 2(e) $\begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$

3(a) $a = -5$ or 3 3(b) $C_A(x) = x^2 - 2x - 3$ 3(c) -4

4. $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}, P = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{pmatrix}$

MAT 188H1F Linear Algebra TERM TEST

Tuesday, November 11, 2003 Duration: 50 minutes

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [9 marks] Parts (a) and (b) are unrelated.

   (a) [5 marks] Find $\det \begin{pmatrix} 0 & 3 & 2 & 1 \\ 1 & 4 & -1 & 1 \\ 0 & 2 & 1 & 1 \\ -1 & 3 & 5 & 6 \end{pmatrix}$

   (b) [4 marks] Find all values of a for which the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & a & 0 \\ 0 & 6 & a \end{pmatrix}$ is *not* invertible.

2. [8 marks; 4 marks for each part] Parts (a) and (b) are unrelated.

   (a) Use Cramer's Rule to find the value of x in the solution to the system of equations $\begin{cases} x + 2y - 3z = 4 \\ y + z = 0 \\ 2x + y - z = 0 \end{cases}$

   (b) Let A and B be 3×3 matrices such that $\det A = 2$ and $\det B = -1$. Find the value of

$$\det\left(A^2 B^6 A^{-1} B^T\right)$$

3. [8 marks] Let $S = \{(1,0,1,-1), (2,1,1,1), (-1,-1,0,-2), (5,1,4,-2)\}$.
   Let $W = \text{span}\,(S)$. Find a basis for each of W and $W^\perp$.

4. [6 marks; 2 marks each] Find the standard matrices of the following linear operators on R^2.

   (a) S is a reflection in the line $y = -x$.

   (b) T is a dilation with scaling factor 3.

   (c) U is an orthogonal projection onto the line $y = x$.

5. [8 marks] Find the eigenvalues of $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ and a basis for each eigenspace of A.

6. [6 marks; one mark each] Indicate whether each of the following statements is True or False. There is no need to justify your answer.

(a) $\{(1,2,0),(1,-1,2),(3,3,5),(-1,3,7)\}$ is a linearly independent set of vectors in R^3.

(b) $\{(1,2,0),(-1,-2,0)\}$ is a linearly independent set of vectors in R^3.

(c) If A is an orthogonal $n\times n$ matrix, then $\det A = \pm 1$.

(d) If $\lambda = 0$ is an eigenvalue of A, then the row vectors of A are linearly independent.

(e) If $T : R^2 \to R^2$ is a reflection in the line $y = mx$, then T is onto.

(f) If A is a symmetric matrix, then $\ker(T_A) = (\text{ran }(T_A))^{\perp}$.

ANSWERS: 1(a) 4 1(b) $a = -3$ or 2 2.(a) $x = -1$ 2(b) -2

3. A basis for W is $\{(1,0,1,-1),(0,1,-1,3)\}$; for $W^{\perp}$, $\{(-1,1,1,0),(1,-3,0,1)\}$

4.(a) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ 4(b) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ 4(c) $\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

5. $\lambda_1 = 1$; basis for eigenspace is $\{(1,0,0),(0,1,1)\}$

$\lambda_2 = -1$; basis for eigenspace is $\{(0,1,-1)\}$

6(a) False (b) False (c) True (d) False (e) True (f) True

MAT 188H1F Linear Algebra TERM TEST

Friday, November 14, 2003 Duration: 50 minutes

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [9 marks] Parts (a) and (b) are unrelated.

   (a) [5 marks] Find det $\begin{pmatrix} 4 & 1 & -1 & 1 \\ 3 & 0 & 2 & 1 \\ 2 & 0 & 1 & 1 \\ 3 & -1 & 5 & 6 \end{pmatrix}$

   (b) [4 marks] Find all values of a for which the matrix $A = \begin{pmatrix} -1 & a & 0 \\ 1 & 1 & 1 \\ 0 & 6 & a \end{pmatrix}$ is *not* invertible.

2. [8 marks; 4 marks for each part] Parts (a) and (b) are unrelated.

   (a) Use Cramer's Rule to find the value of z in the solution to the system of equations $\begin{cases} x + 2y - 3z = 4 \\ \quad\;\; y + z = 0 \\ 2x + y - z = 0 \end{cases}$

   (b) Let A and B be 3×3 matrices such that $\det A = 2$ and $\det B = -1$. Find the value of
   $$\det\left(2A^3 B^T A^{-1}\right)$$

3. [8 marks] Let $S = \{(1, 0, 1, -1), (2, 1, 1, 1), (-1, -1, 0, -2), (5, 1, 4, -2)\}$.
   Let $W = \text{span}\,(S)$. Find a basis for each of W and $W^{\perp}$.

4. [6 marks; 2 marks each] Find the standard matrices of the following linear operators on R^2.

   (a) S is a rotation of 45 degrees, counterclockwise, about the origin.

   (b) T is a dilation with scaling factor 2.

   (c) U is an orthogonal projection onto the line $y = -x$.

5. [8 marks] Find the eigenvalues of $A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{pmatrix}$ and a basis for each eigenspace of A.

6. [6 marks; one mark each] Indicate whether each of the following statements is True or False. There is no need to justify your answer.

(a) $\{(1,2,0),(1,-1,2),(3,3,5),(-1,3,7)\}$ is a spanning set of vectors for R^3.

(b) $\{(1,2,0),(3,-5,0)\}$ is a spanning set of vectors for R^3.

(c) If A is a 3×3 matrix such that $A^T = -A$, then $\det A = 0$.

(d) If A is an $n\times n$ matrix such that row (A) $=$ col (A), then A is a symmetric matrix.

(e) The composition of two reflections about lines through the origin of R^2 ia a rotation about the origin.

(f) If $\mathbf{u}$ and $\mathbf{v}$ are non-zero vectors in R^3, then $\{\mathbf{u},\mathbf{v},\mathbf{u}\times\mathbf{v}\}$ is a basis for R^3.

ANSWERS: 1(a) 4 1(b) $a=-3$ or 2 2.(a) $z=-1$ 2(b) -32

3. A basis for W is $\{(1,0,1,-1),(0,1,-1,3)\}$; for $W^\perp$, $\{(-1,1,1,0),(1,-3,0,1)\}$

4.(a) $\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ 4(b) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ 4(c) $\begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$

5. $\lambda_1 = -2$; basis for eigenspace is $\{(1,0,0),(0,-2,1)\}$

$\lambda_2 = 2$; basis for eigenspace is $\{(0,2,1)\}$

6(a) True (b) False (c) True (d) False (e) True (f) False

University of Toronto

MAT 188H1F Linear Algebra TERM TEST

Tuesday, October 7, 2003

Duration: 50 minutes

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [8 marks] Find the reduced row echelon form of the augmented matrix of the system of equations

$$\begin{array}{rcrcrcrcr} x_1 & + & 3x_2 & + & x_3 & - & x_4 & = & 2 \\ & & x_2 & - & x_3 & + & 2x_4 & = & -3 \\ 3x_1 & + & 13x_2 & - & x_3 & + & 5x_4 & = & -6 \end{array}$$

and then solve the system.

2. [8 marks; 4 marks for each part] Find the following:

   (a) the cosine of the angle between the two vectors

$$\mathbf{u} = (1, 2, -3, 0) \text{ and } \mathbf{v} = (5, 1, 2, -2).$$

   (b) the equation of the plane that is parallel to the vector $(2, 1, -1)$ and contains the line with vector equation

$$(x, y, z) = (0, 1, 0) + t(1, 1, 1),$$

   where t is a parameter. Put your answer in the form $ax + by + cz = d$.

3. [7 marks] Find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ and use it to solve the system of equations

$$\begin{array}{rcrcrcr} x & & & + & z & = & 4 \\ & & y & + & z & = & 8 \\ x & + & y & & & = & -6 \end{array}$$

4. [7 marks] Find the following:

   (a) [4 marks] two elementary matrices E and F such that $B = FEA$, if

$$A = \begin{pmatrix} 1 & 4 & -1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 4 & -1 \\ 0 & -5 & 3 \\ 0 & -3 & 3 \end{pmatrix}$$

(b) [3 marks] conditions on b_1, b_2 and b_3 that ensure the following system is consistent.

$$\begin{array}{rcrcrcl} x & + & y & + & z & = & b_1 \\ x & - & y & + & z & = & b_2 \\ 5x & + & y & + & 5z & = & b_3 \end{array}$$

5. [8 marks; 4 marks each.] Let

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}, C = \begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}, D = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}.$$

(a) Find

$$(AB)^{-1}\left(AC^{-1}\right)\left(D^{-1}C^{-1}\right)^{-1}D^{-1}$$

by first simplifying it as much as possible.

(b) Solve for X if

$$2B + (A - X)^T = C.$$

6. [7 marks]

(a) [4 marks] Let $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Show that $A^2 - 5A + 4I = O$.

(b) [3 marks] Suppose A is any square matrix such that $A^2 - 5A + 4I = O$. Find a formula for A^{-1} in terms of A and I.

ANSWERS: 1. $\left(\begin{array}{cccc|c} 1 & 0 & 4 & -7 & 11 \\ 0 & 1 & -1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$; $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 11 - 4s + 7t \\ -3 + s - 2t \\ s \\ t \end{pmatrix}$, where s and t are parameters.

2.(a) $1/\sqrt{476}$ 2(b) $2x - 3y + z = -3$

3. $A^{-1} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$; $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 4 \\ 8 \\ -6 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ 9 \end{pmatrix}$

4.(a) $E = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $F = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ 4(b) $b_3 = 2b_2 + 3b_1$

5.(a) $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$ 5(b) $\begin{pmatrix} -2 & 1 \\ 2 & 5 \end{pmatrix}$ 6(b) $A^{-1} = -\frac{1}{4}A + \frac{5}{4}I$

University of Toronto

MAT 188H1F Linear Algebra TERM TEST

Friday, October 10, 2003

Duration: 50 minutes

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [8 marks] Find the reduced row echelon form of the augmented matrix of the system of equations

$$\begin{array}{rcrcrcrcr} x_1 & + & 3x_2 & + & x_3 & - & x_4 & = & 1 \\ & & x_2 & - & x_3 & + & 2x_4 & = & -2 \\ 3x_1 & + & 13x_2 & - & x_3 & + & 5x_4 & = & -5 \end{array}$$

and then solve the system.

2. [8 marks; 4 marks for each part] Find the following:

(a) the cosine of the angle between the two vectors

$$\mathbf{u} = (1, -2, 3, 0) \text{ and } \mathbf{v} = (2, -1, 2, 5).$$

(b) the equation of the plane that contains the three points with coordinates

$$(1, 1, 2), (0, 2, 2) \text{ and } (3, 1, 1).$$

Put your answer in the form $ax + by + cz = d$.

3. [7 marks] Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$ and use it to solve the system of equations

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 1 \\ 2x & + & 5y & + & 3z & = & -1 \\ x & & & + & 8z & = & 1 \end{array}$$

4. [7 marks] Find the following:

(a) [4 marks] two elementary matrices E and F such that $B = FEA$, if

$$A = \begin{pmatrix} 2 & 4 & -6 \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & -3 \\ 0 & -2 & 10 \\ 1 & 1 & 2 \end{pmatrix}$$

(b) [3 marks] conditions on b_1, b_2 and b_3 that ensure the following system is consistent.

$$\begin{array}{rcrcrcl} x & + & 2y & + & z & = & b_1 \\ x & + & y & - & z & = & b_2 \\ x & + & 3y & + & 3z & = & b_3 \end{array}$$

5. [8 marks; 4 marks each.] Let

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}, C = \begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}, D = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}.$$

(a) Find

$$\left(CB^{-1}\right)^{-1} (CD)\,(AD)^{-1}\,A^2$$

by first simplifying it as much as possible.

(b) Solve for X if

$$2C + (X - A)^T = D.$$

6. [7 marks]

(a) [4 marks] Let $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Show that $A^2 - 5A + 4I = O$.

(b) [3 marks] Suppose A is any square matrix such that $A^2 - 5A + 4I = O$. Find a formula for A^3 in terms of A and I.

ANSWERS: 1. $\left(\begin{array}{cccc|c} 1 & 0 & 4 & -7 & 7 \\ 0 & 1 & -1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$; $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 7 - 4s + 7t \\ -2 + s - 2t \\ s \\ t \end{pmatrix}$, where s and t are parameters.

2.(a) $5/\sqrt{119}$ 2(b) $x + y + 2z = 6$

3. $A^{-1} = \begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix}$; $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -47 \\ 15 \\ 6 \end{pmatrix}$

4.(a) $E = \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $F = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 4(b) $b_3 = 2b_1 - b_2$

5.(a) $\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$ 5(b) $\begin{pmatrix} -3 & 5 \\ 0 & 0 \end{pmatrix}$ 6(b) $A^3 = 21A - 20I$

University of Toronto
Faculty of Engineering
MAT 188H1F TERM TEST
FRIDAY, NOVEMBER 16, 2001, 9:10 AM
Duration: 50 minutes

Aids Allowed: A non-programmable calculator, to be supplied by student.

Instructions: Fill in the information on this page, and make sure this test contains 4 pages. Present your **solutions** in the space provided. Use the back of the preceding page if you need more space. The value for each question is indicated in square brackets beside each question number.

TOTAL MARKS: 40

1. [15 marks] Assume that the row-reduced echelon form of the matrix

$$A = \begin{pmatrix} 2 & 4 & -1 & 5 & 0 \\ -1 & -2 & 0 & -3 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 6 & -1 \end{pmatrix} \text{ is } R = \begin{pmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

   Find the dimension of, and a basis for, each of the following subspaces:

   (a) the row space of A.

   (b) the column space of A.

   (c) the solution space of $A\mathbf{x} = \mathbf{0}$, where $\mathbf{x}$ is in $\mathbf{R}^5$.

2. [10 marks] Let W be the subset of $\mathbf{M}^{2,2}$ consisting of all 2×2 matrices, the sum of whose entries is zero.

   (a) Show that W is a subspace of $\mathbf{M}^{2,2}$.

   (b) Find a basis for W, and its dimension.

3. [15 marks; 3 marks each] Determine if each of the following statements is True or False, and give a *brief* justification for your choice.

   (a) The set $\{(2,0,1,-1),(0,1,0,5),(6,2,3,7)\}$ is a linearly independent set in $\mathbf{R}^4$.

   (b) The subset W of $\mathbf{M}^{3,3}$ consisting of all 3×3 non-invertible matrices is a subspace of $\mathbf{M}^{3,3}$.

   (c) If the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a basis of a vector space V, then so is the set $\{\mathbf{u}, \mathbf{u}+\mathbf{v}, \mathbf{u}+\mathbf{v}+\mathbf{w}\}$.

   (d) The set of polynomials $\{1+x, 1-x, 2+x^2, x^3+x^4\}$ is a spanning set of P_4.

   (e) Any seven matrices in $\mathbf{M}^{2,3}$ must be linearly dependent.

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MAT 188H1F TERM TEST
FRIDAY, NOVEMBER 16, 2001, 11:10 AM
Duration: 50 minutes

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TOTAL MARKS: 40

1. [15 marks] Assume that the row-reduced echelon form of the matrix

$$A = \begin{pmatrix} 2 & 4 & -1 & 5 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 2 & 3 & 6 & -1 \\ -1 & -2 & 0 & -3 & 2 \end{pmatrix} \text{ is } R = \begin{pmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Find the dimension of, and a basis for, each of the following subspaces:

(a) the row space of A.

(b) the column space of A.

(c) the solution space of $A\mathbf{x} = \mathbf{0}$, where $\mathbf{x}$ is in $\mathbf{R}^5$.

2. [10 marks] Let W be the subset of P_3 consisting of all polynomials $p(x)$ in P_3 such that $p(1) = 0$.

(a) Show that W is a subspace of P_3.

(b) Find a basis for W, and its dimension.

3. [15 marks; 3 marks each] Determine if each of the following statements is True or False, and give a *brief* justification for your choice.

(a) The set $\{(2, 0, 1, -1), (10, -1, 5, -10), (0, 1, 0, 5)\}$ is a linearly independent set in $\mathbf{R}^4$.

(b) The plane in $\mathbf{R}^3$ with equation $x + 2y + 3z = 4$ is a subspace of $\mathbf{R}^3$.

(c) The dimension of the subspace in $\mathbf{R}^4$ consisting of all vectors of the form $(a + b, 2a + 2b, 0, -a - b)$ is 2.

(d) The set of matrices

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right\}$$

is a spanning set of $\mathbf{M}^{2,2}$.

(e) Any five cubic polynomials must be linearly dependent.

University of Toronto
Faculty of Engineering
MAT 188H1F TERM TEST
FRIDAY, NOVEMBER 15, 11:10 AM, 2002
Duration: 50 minutes

Aids Allowed: A non-programmable calculator, to be supplied by student.

Instructions: Fill in the information on this page, and make sure this test contains 4 pages. Present your **solutions** in the space provided. Use the back of the preceding page if you need more space. The value for each question is indicated in square brackets beside each question number. **TOTAL MARKS: 40**

1. [14 marks] Assume that the row-reduced echelon form of the matrix

$$A = \begin{pmatrix} 1 & 1 & 4 & 4 & 3 \\ -1 & 1 & 2 & -6 & 3 \\ 2 & 0 & 2 & 10 & 0 \\ 1 & 2 & 7 & 3 & 2 \end{pmatrix} \text{ is } R = \begin{pmatrix} 1 & 0 & 1 & 5 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Find the following:

(a) [4 marks]

(i) the rank of A. Answer: ________

(ii) the dimension of the row space of A. Answer: ________

(iii) the dimension of the column space of A. Answer: ________

(iv) the dimension of the solution space of the system $A\mathbf{x} = \mathbf{0}$. Answer: ________

(b) [3 marks] a basis for the row space of A.

(c) [3 marks] a basis for the column space of A.

(d) [4 marks] a basis for the solution space of the system $A\mathbf{x} = \mathbf{0}$.

2. [14 marks] Determine if each of the following statements is True or False, and give a *brief* justification for your choice.

(a) [2 marks] $\{(1, 2, 3, 0), (0, -2, 3, 4)\}$ is a spanning set of $\mathbf{R}^4$.

(b) [2 marks] $\{(1, 3, 4), (6, 5, 2), (-1, 7, 3), (4, 3, 6)\}$ is a linearly independent subset of $\mathbf{R}^3$.

(c) [2 marks] $\{(1, 3, 4), (1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is a spanning set of $\mathbf{R}^3$.

(d) [3 marks] The plane with equation $2x + y - 4z = 2$ is a subspace of $\mathbf{R}^3$.

(e) [3 marks] $\{(1, 3, 4), (6, 5, 2), (4, -1, -6)\}$ is a linearly independent subset of $\mathbf{R}^3$.

(f) [2 marks] The subspace of $\mathbf{R}^4$ consisting of all vectors of the form

$$(a+b, 2a+2b, -a-b, 4a+4b)$$

has dimension 2.

3. [12 marks; 4 marks each] Determine if the given set W is a subspace of the given vector space V :

   (a) W is the set of all 2×2 matrices the sum of whose entries is zero; $V = \mathbf{M}^{2,2}$.

   (b) W is the set of all 2×2 matrices whose determinant is zero; $V = \mathbf{M}^{2,2}$.

   (c) W is the subset of $\mathbf{P}_2$ consisting of all polynomials $\mathbf{p}$ satisfying $\mathbf{p}(1) = 0$; $V = \mathbf{P}_2$.

ANSWERS: 1(a)(i) 3   1(a)(ii) 3   1(a)(iii) 3   1(a)(iv) 2

1(b) any 3 independent rows of A or R will do. For example,

$\{(1,1,4,4,3), (-1,1,2,-6,3), (1,2,7,3,2)\}$ or $\{(1,0,1,5,0), (0,1,3,-1,0), (0,0,0,0,1)\}$

1(c)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 0 \\ 2 \end{pmatrix} \right\}$$

1(d)

$$\left\{ \begin{pmatrix} -1 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

2.(a) F   (b) F   (c) T   (d) F   (e) F   (f) F

3(a) W is a subspace   (b) W is not a subspace   (c) W is a subspace

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Faculty of Engineering
MAT 188H1F TERM TEST
FRIDAY, NOVEMBER 15, 12:10 PM, 2002
Duration: 50 minutes

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1. [14 marks] Assume that the row-reduced echelon form of the matrix

$$A = \begin{pmatrix} 1 & 1 & 4 & 4 & 3 \\ -1 & 1 & 2 & -6 & 3 \\ 2 & 0 & 2 & 10 & 0 \\ 1 & 2 & 7 & 3 & 2 \end{pmatrix} \text{ is } R = \begin{pmatrix} 1 & 0 & 1 & 5 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Find the following:

(a) [4 marks]

(i) the rank of A. Answer: _______

(ii) the dimension of the row space of A. Answer: _______

(iii) the dimension of the column space of A. Answer: _______

(iv) the dimension of the solution space of the system $A\mathbf{x} = \mathbf{0}$. Answer: _______

(b) [3 marks] a basis for the row space of A.

(c) [3 marks] a basis for the column space of A.

(d) [4 marks] a basis for the solution space of the system $A\mathbf{x} = \mathbf{0}$.

2. [14 marks] Determine if each of the following statements is True or False, and give a *brief* justification for your choice.

(a) [2 marks] $\{(1,2,3,0),(0,-2,3,4)\}$ is a spanning set of $\mathbf{R}^4$.

(b) [2 marks] $\{(1,3,4),(6,5,2),(-1,7,3),(4,3,6)\}$ is a linearly independent subset of $\mathbf{R}^3$.

(c) [2 marks] $\{(1,3,4),(1,0,0),(0,1,0),(0,0,1)\}$ is a spanning set of $\mathbf{R}^3$.

(d) [3 marks] The plane with equation $2x + y - 4z = 0$ is a subspace of $\mathbf{R}^3$.

(e) [3 marks] $\{(1,3,4),(6,5,2),(4,-1,-6)\}$ is a linearly independent subset of $\mathbf{R}^3$.

(f) [2 marks] The subspace of $\mathbf{R}^4$ consisting of all vectors of the form

$$(a + 2c, b + d, 2a + 4c, -b - d)$$

has dimension 2.

3. [12 marks; 4 marks each] Determine if the given set W is a subspace of the given vector space V :

   (a) W is the set of all 2×2 matrices whose entries on the main diagonal add up to zero; $V = \mathbf{M}^{2,2}$.

   (b) W is the set of 2×2 invertible matrices; $V = \mathbf{M}^{2,2}$.

   (c) W is the set of polynomials with degree less than or equal to three, the sum of whose coefficients is zero; $V = \mathbf{P}_3$.

ANSWERS: 1(a)(i) 3 1(a)(ii) 3 1(a)(iii) 3 1(a)(iv) 2

1(b) any 3 independent rows of A or R will do. For example,

$\{(1, 1, 4, 4, 3), (-1, 1, 2, -6, 3), (1, 2, 7, 3, 2)\}$ or $\{(1, 0, 1, 5, 0), (0, 1, 3, -1, 0), (0, 0, 0, 0, 1)\}$

1(c)

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 0 \\ 2 \end{pmatrix} \right\}$$

1(d)

$$\left\{ \begin{pmatrix} -1 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

2.(a) F (b) F (c) T (d) T (e) F (f) T

3(a) W is a subspace (b) W is not a subspace (c) W is a subspace