University of Toronto

MAT186H1F TERM TEST

WEDNESDAY, NOVEMBER 9, 2005, 5:10 PM

Duration: 50 minutes

Aids Allowed: Casio 260, Sharp 520 or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [12 marks; 4 for each part] Find the following:

   (a) the value of $\dfrac{dy}{dx}$ at the point $(x, y) = (1, 1)$ if $(x^2 + y^2)^2 = 4xy$.

   (b) the linear approximation to $\sqrt[4]{15}$.

   (c) $F(x)$, if $F'(x) = \dfrac{1}{x^2} + e^{-x}$ and $F(-1) = 2e$.

2. [10 marks; 5 for each part] Find the following limits:

   (a) $\lim\limits_{x\to 0} \dfrac{1 - \sec x}{e^x - 1 - x}$

   (b) $\lim\limits_{x\to\infty} \left(\cos\left(\dfrac{1}{\sqrt{x}}\right)\right)^x$

3. [13 marks] Let $f(x) = \dfrac{10 \ln x}{x^2}$; you may assume

$$f'(x) = \frac{10 - 20 \ln x}{x^3}; f''(x) = \frac{60 \ln x - 50}{x^4}.$$

   (a) [3 marks] Find the open intervals on which f is increasing; decreasing.

   (b) [3 marks] Find the open intervals on which f is concave up; concave down.

   (c) [3 marks] Find both $\lim\limits_{x\to 0^+} f(x)$ and $\lim\limits_{x\to\infty} f(x)$

   (d) [4 marks] Sketch the graph of f, labelling all critical points, inflection points and asymptotes, if any.

4. [10 marks] The volume of a rectangular box is to be 972 cm^3. The base of the box is to be a rectangle with one side twice as long as the other. The box will be open at the top. Find the dimensions of the box that will minimize its total surface area.

ANSWERS: 1(a) -1 1(b) $\dfrac{63}{32}$ 1(c) $F(x) = -\dfrac{1}{x} - e^{-x} + 3e - 1$ 2(a) -1 2(b) $e^{-1/2}$

3(a) f increasing on $(0, \sqrt{e})$; f decreasing on $(\sqrt{e}, \infty)$; global max at $(\sqrt{e}, 5/e)$

3(b) f concave up on $(e^{5/6}, \infty)$; f concave down on $(0, e^{5/6})$; inflection point at $\left(e^{5/6}, \dfrac{25}{3e^{5/3}}\right)$

3(c) $\lim\limits_{x\to 0^+} \dfrac{10 \ln x}{x^2} = -\infty$ and $\lim\limits_{x\to\infty} \dfrac{10 \ln x}{x^2} = 0$

3(d) above details plus: vertical asymptote at $x = 0$ and horizontal asymptote at $y = 0$

4. 9 cm $\times$ 18 cm $\times$ 6 cm

University of Toronto

MAT 186H1F TERM TEST

WEDNESDAY, OCTOBER 5, 2005, 5:10 PM

Duration: 50 minutes

Aids Allowed: Casio 260, Sharp 520 or Texas Instrument 30 calculator.
Instructions: DO NOT USE L'Hopital's Rule on this test.
TOTAL MARKS: 45

1. [10 marks] Given that $\sin x = \dfrac{2}{3}$ and that $\cos x < 0$, find the exact values of each of the following:

   (a) [3 marks] $\cos x$

   (b) [4 marks] $\sin\left(x + \dfrac{\pi}{3}\right)$

   (c) [3 marks] $\cos(2x)$

2. [15 marks] Find the following limits, or explain why they do not exist.

   (a) [3 marks] $\lim\limits_{w\to 2} \dfrac{w^3 - 8}{w - 2}$

   (b) [4 marks] $\lim\limits_{x\to -5} \dfrac{x+5}{|x+5|}$

   (c) [4 marks] $\lim\limits_{h\to 0} \dfrac{\sqrt{4+h} - 2}{h}$

   (d) [4 marks] $\lim\limits_{x\to 0} (x^4 + 3x^2)\cos\left(\dfrac{1}{x}\right)$

3(a) [4 marks] Find the equation of the normal line to the graph of

$$f(x) = \sqrt{x^4 + 3}$$

at the point $(x, y) = (-1, 2)$.

3(b) [7 marks] Find the maximum and minimum values of $f(x) = 5x^{2/3} - x^{5/3}$ on the closed interval $[-1, 4]$.

4(a) [4 marks] Find $\lim\limits_{h\to 0} \dfrac{\cos(x+h) - \cos(x-h)}{h}$.

4(b) [5 marks] Let $f(x) = x^2 + 6x + 1$. Find all the points on the graph of $y = f(x)$ from which the tangent lines to $y = f(x)$ pass through the origin $(0, 0)$.

ANSWERS: 1.(a) $-\sqrt{5}/3$ (b) $(2 - \sqrt{15})/6$ (c) $1/9$
2.(a) 12 (b) does not exist (c) $1/4$ (d) 0
3.(a) $y = x + 3$ (b) max: 6 min: 0
4.(a) $-2\sin x$ (b) $(1, 8)$ and $(-1, -4)$

University of Toronto

MAT 186H1F Calculus I TERM TEST

Monday, November 1, 2004, 5:10 PM

1. [13 marks] Find the following:

   (a) [4 marks] $\dfrac{d}{dx}\sqrt{\sec\sqrt{x}}$

   (b) [4 marks] $\dfrac{d}{dx}\left(x^3 e^{\cos x}\right)$

   (c) [5 marks] $\dfrac{dy}{dx}$ at $(x, y) = (1, -1)$ if

   $$e^y \ln x = \sin(x + y)$$

2. [12 marks; 4 for each part] The parts of this question are unrelated.

   (a) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = x^{1/x}$

   (b) Let $f(x) = x^2 - 5$. Use Newton's method to approximate $\sqrt{5}$. Start with $x_0 = 2$ and calculate x_1 and x_2. Give your answers to 5 decimal places.

   (c) Find the equation of the slant asymptote to the graph of

   $$f(x) = \frac{x^3 + 2x^2 - 3}{x^2}$$

3. [12 marks] It is a short ferry ride from Cormorant Island to the city. A company operates a small fleet of ferrys to transport commuters between the city and the island. At present, each ferry makes six trips per day carrying forty passengers, who each pay one dollar fare.

   The company wants to maximize its revenue per ferry per day. After some market research, they find that for each extra trip that each ferry makes per day, 2 extra passengers will travel on each trip if the fare is decreased by 5 cents for each extra trip.

   (a) [3 marks] Explain briefly why the function that the company needs to maximize is

   $$f(x) = (6 + x)(40 + 2x)(1 - .05x)$$

   where x is the number of extra trips each ferry makes per day.

   (b) [2 marks] In the context of this problem, find the closed interval on which $f(x)$ should be maximized.

   (c) [7 marks] What should the fare for each trip be to maximize total daily revenue?

4. [13 marks] Let $f(x) = \dfrac{1}{x^2 - 2x}$. You may assume that

$$f'(x) = \frac{2(1-x)}{(x^2-2x)^2} \text{ and } f''(x) = \frac{2(3x^2-6x+4)}{(x^2-2x)^3}$$

(a) [4 marks] Find the intervals on which f is increasing; decreasing.

(b) [3 marks] Find the intervals on which f is concave up; concave down.

(c) [6 marks] Plot the graph of $y = f(x)$ labelling all critical points, all inflection points and all asymptotes, if any.

ANSWERS:

1.(a) $\dfrac{1}{2\sqrt{\sec\sqrt{x}}} \cdot \sec\sqrt{x}\tan\sqrt{x} \cdot \dfrac{1}{2\sqrt{x}}$ (b) $3x^2e^{\cos x} + x^3e^{\cos x}(-\sin x)$ (c) $e^{-1} - 1$

2(a) $\dfrac{x^{1/x}}{x^2}(1 - \ln x)$ (b) $x_1 = 2.25; x_2 = 2.23611$ (c) $y = x + 2$

3(a) Explain what each factor represents:

- $6 + x$ is the number of trips per ferry per day
- $40 + 2x$ is the number of passengers per trip
- $1 - .05x$ is the fare per passenger in dollars

So $f(x) = (6 + x)(40 + 2x)(1 - .05x)$ is the revenue per ferry per day.

3(b) $[-6, 20]$ (c) 50 cents

4(a) f increasing on $(-\infty, 0)$ and $(0, 1)$; decreasing on $(1, 2)$ and $(2, \infty)$

4(b) f is concave up on $(-\infty, 0)$ and $(2, \infty)$; concave down on $(0, 2)$

4(c) Relative maximum point at $(1, -1)$; no inflection points. Vertical asymptotes at $x = 0$ and $x = 2$. Horizontal asymptote at $y = 0$ on both sides of the graph.

MAT 186H1F Calculus I TERM TEST
Monday, October 4, 2004, 5:10-6 PM

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.
Do not use L'Hôpital's Rule on this test.

1. [25 marks] Evaluate the following limits or explain why they do not exist.

   (a) [4 marks] $\lim_{x\to 5} \dfrac{x^2-25}{x^2-3x-10}$

   (b) [4 marks] $\lim_{t\to 0} \dfrac{3t}{2-\sqrt{4+t}}$

   (c) [3 marks] $\lim_{x\to 3} \dfrac{x^2-9}{|x^2-9|}$

   (d) [2 marks] $\lim_{\theta\to 0} \dfrac{\tan\theta}{\theta}$

   (e) [3 marks] $\lim_{x\to 0} \dfrac{1}{x}\sin\left(\dfrac{x}{3}\right)$

   (f) [3 marks] $\lim_{x\to 0} x^2\tan\left(\dfrac{x}{3}\right)$

   (g) [3 marks] $\lim_{x\to 0} x\sin\left(\dfrac{3}{x}\right)$

   (h) [3 marks] $\lim_{x\to 4^+} \dfrac{x-5}{x^2-16}$

2. [9 marks] For both of the following equations, find all the exact solutions in the interval $[0,\pi]$.

   (a) [4 marks] $2\cos^2 x + 3\sin^2 x = 3$

   (b) [5 marks] $8\sin^2 x\cos^2 x = 1$

3. [4 marks] Apply the Intermediate Value Property to show that the equation

   $$x^6 - x = 1$$

   has at least one solution.

4. [6 marks] Find the equation of the tangent line and of the normal line to the graph of

   $$f(x) = \frac{x^{1/3}+1}{x^2+1}$$

   at the point $(x,y) = (-1,0)$. (Put your answer in $y = mx + b$ form.)

5. [6 marks] An oil supertanker has run aground on an isolated reef in the middle of the Pacific Ocean and is leaking oil. The oil slick surrounding the supertanker is circular in shape and its circumference is growing at a rate of 10 meters per hour.

   At what rate is the area of the oil slick increasing when its circumference is 200π meters?

ANSWERS: 1(a) 10/7 (b) -12 (c) does not exist (d) 1 (e) 1/3 (f) 0 (g) 0 (h) $-\infty$
2(a) $x = \pi/2$ (b) $x = \pi/8$ or $3\pi/8$ or $5\pi/8$ or $7\pi/8$ 3. There is a solution in $(1,2)$
4. tangent: $y = x/6 + 1/6$ normal: $y = -6x - 6$ 5. $dA/dt = 1000$ m^2/hr

MAT 186H1F Calculus I TERM TEST

Monday, November 3, 2003, 2:10 PM Duration: 50 minutes

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [8 marks; 4 marks each.] Parts (a) and (b) of this question are unrelated.

   (a) Approximate the solution to the equation $x^3 + x - 1 = 0$ by using Newton's method. Start with $x_0 = 0.5$ and calculate x_1 and x_2. (Round off your answers to 5 decimal places.)

   (b) Find $\dfrac{dy}{dx}$ at the point $(x, y) = (2, 1)$ if $y = \left(\dfrac{2}{x}\right)^{x^2+4}$.

2. [6 marks] Find the (global) maximum and minimum values attained by the function $f(x) = 3x^5 - 5x^3$ on the closed interval $[-1, 2]$.

3. [8 marks] Let $f(x) = 4x^{1/3} - x^{4/3}$. Find the intervals on which f is increasing; decreasing; concave up; concave down.

4. [6 marks] Find the equations of all the asymptotes (vertical, horizontal or slant) to the graph of $f(x) = \dfrac{x^2 + 3}{x + 1}$.

5. [8 marks; 4 marks each.] Find the following limits, or explain why they do not exist.

   (a) $\lim\limits_{x\to\infty} x^2 e^{-x}$.

   (b) $\lim\limits_{x\to 0} \dfrac{\tan x - x}{x^3}$.

6. [9 marks] Find all the points on the graph of $(x^2+y^2)^2 = y^2 - x^2$ at which the tangent lines to the graph are horizontal or vertical.

ANSWERS: 1(a) $x_1 = 0.71429; x_2 = 0.68318$ 1(b) -4

2. Global max is 56; global min is -2.

3. f is increasing on $(-\infty, 0)$ and $(0, 1)$; f is decreasing on $(1, \infty)$.
f is concave up on $(-2, 0)$; f is concave down on $(-\infty, -2)$ and $(0, \infty)$.

4. Vertical asymptote: $x = -1$; no horizontal asymptotes; slant asymptote: $y = x - 1$

5.(a) 0 5(b) 1/3 6. Horizontal tangents at $(0, \pm 1)$; vertical tangents at $\left(\pm\dfrac{1}{\sqrt{8}}, \pm\sqrt{\dfrac{3}{8}}\right)$

MAT 186H1F Calculus I TERM TEST

Wednesday, November 5, 2003, 2:10 PM Duration: 50 minutes

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [8 marks; 4 marks each.] Parts (a) and (b) of this question are unrelated.

   (a) Approximate the solution to the equation $x^3 - x - 1 = 0$ by using Newton's method. Start with $x_0 = 1.5$ and calculate x_1 and x_2. (Round off your answers to 5 decimal places.)

   (b) Find $\dfrac{dy}{dx}$ at the point $(x, y) = (1, 1)$ if $y = \left(\dfrac{x^2+1}{2}\right)^{\sqrt{x}}$.

2. [6 marks] Find the (global) maximum and minimum values attained by the function $f(x) = x^4 - 2x^2 + 10$ on the closed interval $[-2, 3]$.

3. [8 marks] Let $f(x) = x^{5/3} - 5x^{2/3}$. Find the intervals on which f is increasing; decreasing; concave up; concave down.

4. [6 marks] Find the equations of all vertical or horizontal asymptotes to the graph of $f(x) = \dfrac{2x^2 - x - 1}{x^2 - 1}$.

5. [8 marks; 4 marks each.] Find the following limits, or explain why they do not exist.

   (a) $\displaystyle\lim_{x \to 0^+} \frac{\ln(\sin x)}{\cot x}$

   (b) $\displaystyle\lim_{x \to 0} \frac{2 - e^x - e^{-x}}{x^2}$

6. [9 marks] Find all the points on the graph of $(x^2+y^2)^2 = x^2 - y^2$ at which the tangent lines to the graph are horizontal or vertical.

ANSWERS: 1(a) $x_1 = 1.34783; x_2 = 1.32520$ 1(b) 1

2. Global max is 73; global min is 9.

3. f is increasing on $(-\infty, 0)$ and $(2, \infty)$; f is decreasing on $(0, 2)$.
f is concave up on $(-1, 0)$ and $(0, \infty)$; f is concave down on $(-\infty, -1)$

4. Vertical asymptote: $x = -1$; horizontal asymptote: $y = 2$

5.(a) 0 5(b) -1 6. Vertical tangents at $(\pm 1, 0)$; horizontal tangents at $\left(\pm\sqrt{\dfrac{3}{8}}, \pm\dfrac{1}{\sqrt{8}}\right)$

MAT 186H1F Calculus I TERM TEST

Friday, November 7, 2003, 2:10 PM Duration: 50 minutes

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [8 marks; 4 marks each.] Parts (a) and (b) of this question are unrelated.

   (a) Approximate the solution to the equation $x^3 + 2x - 1 = 0$ by using Newton's method. Start with $x_0 = 0.5$ and calculate x_1 and x_2. (Round off your answers to 5 decimal places.)

   (b) Find $\dfrac{dy}{dx}$ at the point $(x, y) = (0, 1)$ if $y = \left(\dfrac{3 + e^x}{4}\right)^{\cos x}$.

2. [6 marks] Find the (global) maximum and minimum values attained by the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 2$ on the closed interval $[-2, 3]$.

3. [8 marks] Let $f(x) = 8x^{1/3} + 4x^{4/3}$. Find the intervals on which f is increasing; decreasing; concave up; concave down.

4. [6 marks] Find the equations of all vertical or horizonatl asymptotes to the graph of $f(x) = \dfrac{\sqrt{x^2 + 1}}{x}$.

5. [8 marks; 4 marks each.] Find the following limits, or explain why they do not exist.

   (a) $\lim\limits_{x \to 0^+} \dfrac{\ln x}{\ln(\tan x)}$.

   (b) $\lim\limits_{x \to 0} \dfrac{2e^{-x} - x^2 + 2x - 2}{x^3}$.

6. [9 marks] Find all the points on the graph of the "peanut" with equation $(x^2 + y^2)^2 = 5x^2 + y^2$ at which the tangent lines to the graph are horizontal. (Note: disregard the point $(0, 0)$ which satisfies the equation, but is an isolated point not connected to the rest of the graph.)

ANSWERS: 1(a) $x_1 = 0.45455; x_2 = 0.45340$ 1(b) $1/4$

2. Global max is 34; global min is -30

3. f is increasing on $(-1/2, 0)$ and $(0, \infty)$; f is decreasing on $(-\infty, -1/2)$.
f is concave up on $(-\infty, 0)$ and $(1, \infty)$; f is concave down on $(0, 1)$

4. Vertical asymptote: $x = 0$; horizontal asymptotes: $y = 1$ and$y = -1$

5.(a) 1 5(b) $-1/3$ 6. Horizontal tangents at $(0, \pm 1)$ and $\left(\pm\dfrac{\sqrt{15}}{4}, \pm\dfrac{5}{4}\right)$

MAT 186H1F Calculus I TERM TEST
Monday, September 29, 2003, 2:10 PM
Duration: 50 minutes
Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [6 marks] The position x at time t of a particle moving along the x-axis is given by

$$x = -16t^2 + 80t + 40.$$

At which time is the velocity of the particle zero, and what is its position at that time?

2. [8 marks] Let $f(x) = \dfrac{2x}{x^2+1}$. Find the equations of both the tangent line and the normal line to $y = f(x)$ at the point $(x, y) = (2, \frac{4}{5})$.

(Put your answers in $y = mx + b$ form.)

3. [15 marks] Find the following limits, or explain why they do not exist:

(a) [4 marks] $\lim\limits_{t\to -2} \dfrac{t^2 - t - 6}{t^2 + t - 2}$

(b) [4 marks] $\lim\limits_{x\to 0} \dfrac{\sqrt{x+9} - 3}{x}$

(c) [3 marks] $\lim\limits_{x\to 3} \dfrac{|x-3|}{x-3}$

(d) [4 marks] $\lim\limits_{x\to 0} \dfrac{\sin(5x)}{\sin x}$

4. [9 marks; 3 marks each.] Suppose $\sin\theta = \dfrac{3}{4}$ and $\cos\theta < 0$. Use trigonometric identities to find the exact values (*not* decimal approximations) of

(a) $\cos\theta$

(b) $\sin\left(\theta + \dfrac{\pi}{6}\right)$

(c) $\cos(2\theta)$

5. [7 marks] State the Intermediate Value Property, and use it to show that the equation

$$x^3 + x - 1 = 0$$

has a solution in the closed interval $[0, 1]$.

ANSWERS: 1. $v = 0 \Rightarrow t = 2.5 \Rightarrow x = 140$ 2. tangent: $y = -\dfrac{6}{25}x + \dfrac{32}{25}$; normal: $y = \dfrac{25}{6}x - \dfrac{113}{15}$ 3.(a) $\dfrac{5}{3}$ 3.(b) $\dfrac{1}{6}$ 3.(c) does not exist 3(d) 5 4(a). $-\dfrac{\sqrt{7}}{4}$ 4(b) $\dfrac{3\sqrt{3} - \sqrt{7}}{8}$ 4(c) $-\dfrac{1}{8}$ 5. Suppose f is continuous on the closed interval $[a, b]$ and that K is any number between $f(a)$ and $f(b)$. Then there is a number c in (a, b) such that $f(c) = K$.

MAT 186H1F Calculus I TERM TEST
Wednesday, October 1, 2003, 2:10 PM
Duration: 50 minutes
Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [6 marks] The position x at time t of a particle moving along the x-axis is given by

$$x = 16t^2 - 160t + 40.$$

At which time is the velocity of the particle zero, and what is its position at that time?

2. [8 marks] Let $f(x) = \dfrac{x^3}{x+2}$. Find the equations of both the tangent line and the normal line to $y = f(x)$ at the point $(x, y) = (2, 2)$.
(Put your answers in $y = mx + b$ form.)

3. [15 marks] Find the following limits, or explain why they do not exist:

(a) [4 marks] $\lim\limits_{t\to -1} \dfrac{t^2 - t - 2}{t^2 - 3t - 4}$

(b) [4 marks] $\lim\limits_{x\to 5} \dfrac{\frac{1}{x} - \frac{1}{5}}{x - 5}$

(c) [3 marks] $\lim\limits_{x\to 3} \dfrac{x^2 - 9}{|x - 3|}$

(d) [4 marks] $\lim\limits_{x\to 0} \dfrac{\tan x}{\sin 2x}$

4. [9 marks; 3 marks each.] Suppose $\sin\theta = \dfrac{3}{5}$ and $\cos\theta < 0$. Use trigonometric identities to find the exact values (*not* decimal approximations) of

(a) $\cos\theta$

(b) $\sin\left(\theta - \dfrac{\pi}{3}\right)$

(c) $\sin(2\theta)$

5. [7 marks] State the Squeeze Law of Limits, and use it to find the value of

$$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right).$$

ANSWERS: 1. $v = 0 \Rightarrow t = 5 \Rightarrow x = -360$ 2. tangent: $y = \dfrac{5}{2}x - 3$; normal: $y = -\dfrac{2}{5}x - \dfrac{14}{5}$ 3.(a) $\dfrac{3}{5}$ 3.(b) $-\dfrac{1}{25}$ 3.(c) does not exist 3(d) $\dfrac{1}{2}$ 4(a). $-\dfrac{4}{5}$ 4(b) $\dfrac{3+4\sqrt{3}}{10}$ 4(c) $-\dfrac{24}{25}$ 5. Suppose $f(x) \le g(x) \le h(x)$ for all $x \neq a$ in some neighbourhood of a such that $\lim\limits_{x\to a} f(x) = L = \lim\limits_{x\to a} h(x)$. Then $\lim\limits_{x\to a} g(x) = L$. $\lim\limits_{x\to 0} x^2 \cos\left(\dfrac{1}{x}\right) = 0$.

MAT 186H1F Calculus I TERM TEST
Friday, October 3, 2003, 2:10 PM
Duration: 50 minutes
Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

TOTAL MARKS: 45

1. [6 marks] The position x at time t of a particle moving along the x-axis is given by

$$x = 16t^2 - 160t + 80.$$

At which time is the velocity of the particle zero, and what is its position at that time?

2. [8 marks] Let $f(x) = \dfrac{x-3}{x^3+1}$. Find the equations of both the tangent line and the normal line to $y = f(x)$ at the point $(x, y) = (1, -1)$.
(Put your answers in $y = mx + b$ form.)

3. [15 marks] Find the following limits, or explain why they do not exist:

(a) [4 marks] $\lim\limits_{t\to 2} \dfrac{t^2+t-6}{t^2-t-2}$

(b) [4 marks] $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{x+4}-2}$

(c) [3 marks] $\lim\limits_{x\to -2} \dfrac{|2+x|}{x+2}$

(d) [4 marks] $\lim\limits_{x\to 0} \dfrac{x+\sin(5x)}{\sin x}$

4. [9 marks; 3 marks each.] Suppose $\sin\theta = \dfrac{1}{4}$ and $\cos\theta < 0$. Use trigonometric identities to find the exact values (*not* decimal approximations) of

(a) $\cos\theta$

(b) $\sin\left(\theta - \dfrac{\pi}{4}\right)$

(c) $\sin(2\theta)$

5. [7 marks] State what it means for the function $f(x)$ to be continuous at $x = a$. Then for the function

$$f(x) = \frac{x+3}{x^2-9}$$

find all its discontinuities and determine if any of them is removable.

ANSWERS: 1. $v = 0 \Rightarrow t = 5 \Rightarrow x = -320$ 2. tangent: $y = 2x - 3$; normal: $y = -\dfrac{1}{2}x - \dfrac{1}{2}$ 3.(a) $\dfrac{5}{3}$ 3.(b) 4 3.(c) does not exist 3(d) 6 4(a). $-\dfrac{\sqrt{15}}{4}$ 4(b) $\dfrac{1+\sqrt{15}}{4\sqrt{2}}$ 4(c) $-\dfrac{\sqrt{15}}{8}$ 5. $f(x)$ is continuous at $x = a$ if $f(a)$ is defined, $\lim\limits_{x\to a} f(x)$ exists, and $\lim\limits_{x\to a} f(x) = f(a)$. Given f has a removable discontinuity at $x = -3$; a non-removable discontinuity at $x = 3$.

University of Toronto
Faculty of Engineering
MAT 186H1F Calculus I TERM TEST
Monday, November 4, 2002, 9:10 AM
Duration: 50 minutes

Aids Allowed: A non-programmable calculator, to be supplied by student.

Instructions: Present your **solutions** in the space provided. The value for each question is indicated in square brackets beside each question number. **TOTAL MARKS: 40**

1. [12 marks] Find the following:

   (a) [3 marks] $\int \cos(3x)\,dx$

   (b) [4 marks] $\int \dfrac{(x-1)^2}{\sqrt{x}}\,dx$

   (c) [5 marks] $\int \sqrt{1-\sqrt{x}}\,dx$

2. [16 marks] Let $f(x) = xe^{-x}$.

   (a) [4 marks] Find the intervals on which f is increasing; decreasing.

   (b) [4 marks] Find the intervals on which f is concave up; concave down.

   (c) [4 marks] Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

   (d) [4 marks] Sketch the graph of $y = f(x)$, indicating all critical points, all inflection points, and all horizontal asymptotes, if any.

3. [12 marks] Find the following limits:

   (a) [3 marks] $\lim_{x\to 0} \dfrac{\sin(3x)}{x}$

   (b) [4 marks] $\lim_{x\to 0} (1+2x)^{\csc x}$

   (c) [5 marks] $\lim_{x\to\infty} \left(x\left(1+\dfrac{2}{x}+\dfrac{5}{x^2}\right)^{1/5} - x \right)$

ANSWERS:

1.(a) $\dfrac{1}{3}\sin(3x)+c$ (b) $\dfrac{2}{5}x^{5/2}-\dfrac{4}{3}x^{3/2}+2\sqrt{x}+c$ (c) $\dfrac{4}{5}(1-\sqrt{x})^{5/2}-\dfrac{4}{3}(1-\sqrt{x})^{3/2}+c$

2.(a) f increasing for $x<1$; decreasing for $x>1$. (b) f is concave up for $x>2$; concave down for $x<2$. (c) $\lim_{x\to\infty} f(x)=0$ and $\lim_{x\to-\infty} f(x)=-\infty$.

(d) Max at $(1,1/e)$; inflection point at $(2,2/e^2)$; HA at $y=0$.

3.(a) 3 (b) e^2 (c) 2/5

University of Toronto
Faculty of Engineering
MAT 186H1F Calculus I TERM TEST
Wednesday, November 6, 2002, 4:10 PM
Duration: 50 minutes

Aids Allowed: A non-programmable calculator, to be supplied by student.

Instructions: Present your **solutions** in the space provided. The value for each question is indicated in square brackets beside each question number. **TOTAL MARKS: 40**

1. [12 marks] Find the following:

   (a) [3 marks] $\int \sec(5x)\tan(5x)\,dx$

   (b) [4 marks] $\int e^{-x}\left(2+e^{-x}\right)^3\,dx$

   (c) [5 marks] $\int \left(1+\sqrt{x}\right)^{3/2}\,dx$

2. [16 marks] Let $f(x) = x^2 \ln x$, for $x > 0$.

   (a) [4 marks] Find the intervals on which f is increasing; decreasing.

   (b) [4 marks] Find the intervals on which f is concave up; concave down.

   (c) [4 marks] Find $\lim_{x\to 0^+} f(x)$.

   (d) [4 marks] Sketch the graph of $y = f(x)$, indicating all critical points and all inflection points, if any.

3. [12 marks] Find the following limits:

   (a) [3 marks] $\lim_{x\to 0} \dfrac{\tan(4x)}{x}$

   (b) [4 marks] $\lim_{x\to 1} \left(\dfrac{x}{\ln x} - \dfrac{1}{x\ln x}\right)$

   (c) [5 marks] $\lim_{x\to\infty} \left(\dfrac{3+x}{x}\right)^{2x+1}$

ANSWERS:

1.(a) $\dfrac{1}{5}\sec(5x)+c$ (b) $-\dfrac{1}{4}(2+e^{-x})^4+c$ (c) $\dfrac{4}{7}(1+\sqrt{x})^{7/2}-\dfrac{4}{5}(1+\sqrt{x})^{5/2}+c$

2.(a) f increasing for $x > e^{-1/2}$; decreasing for $0 < x < e^{-1/2}$.

(b) f is concave up for $x > e^{-3/2}$; concave down for $0 < x < e^{-3/2}$. (c) $\lim_{x\to 0^+} f(x) = 0$.

(d) Min at $(e^{-1/2}, (-1/2)e^{-1})$; inflection point at $(e^{-3/2}, (-3/2)e^{-3})$.

3.(a) 4 (b) 2 (c) e^6

University of Toronto
Faculty of Engineering
MAT 186H1S TERM TEST
THURSDAY, MARCH 8, 2001
Duration: 90 minutes

Aids Allowed: Calculator, to be supplied by student.

Instructions: Present your **solutions** in the booklets provided.

TOTAL MARKS: 40 The value of each question is indicated in parantheses beside the question number.

1. [15 marks] Find the following:
   (a) [5 marks] $\dfrac{dy}{dx}$ at the point $(x, y) = (1, -1)$ if $x^2 \sin(x + y) = y^2 + xy$.
   (b) [5 marks] $\dfrac{dy}{dx}$ if $y = x^{\tan x}$
   (c) [5 marks] x_3 if Newton's method is used to approximate a solution to the equation $x^3 + x - 1 = 0$, starting with $x_1 = 0.5$

2. [15 marks] Let $f(x) = \dfrac{x^2 - 2x + 3}{x^2}$. Find the intervals on which the graph of $y = f(x)$ is increasing, decreasing, concave up, and concave down. Plot the graph of $y = f(x)$, labelling all critical points, all inflection points, and all asymptotes (if any.)

3. [10 marks] A manufacturer builds cylindrical metal cans that hold 1000 cm^3. There is no waste involved in cutting material for the curved surface (sides) of the can. However, each circular end piece (for the top and bottom) is cut from a square piece of metal, leaving four waste pieces. Find the dimensions of the can (height and radius) that use the least amount of metal, including all the waste materials. (Recall: the volume of a cylinder is $V = \pi r^2 h$.)

University of Toronto
Faculty of Engineering
MAT 186H1F TERM TEST
MONDAY, OCTOBER 29, 2001, 9:10 AM
Duration: 50 minutes

Aids Allowed: A non-programmable calculator, to be supplied by student.

Instructions: Fill in the information on this page, and make sure this test contains 4 pages. Present your **solutions** in the space provided. Use the back of the preceding page if you need more space. The value for each question is indicated in square brackets beside each question number.

TOTAL MARKS: 40

1. [15 marks] Let $f(x) = x^3 + 4x - 1$.

   (a) [6 marks] Explain why the equation $f(x) = 0$ has exactly one real solution.

   (b) [9 marks] Use Newton's method to approximate the solution to the equation $f(x) = 0$, correct to 3 decimal places.

2. [15 marks] Let $f(x) = x - 3x^{1/3}$.

   (a) [6 marks] Find all critical points of f and determine if each critical point is a relative maximum point or minimum point.

   (b) [4 marks] On which intervals is the graph of f concave up? concave down?

   (c) [5 marks] Sketch the graph of $y = f(x)$, on the interval $-8 \leq x \leq 8$, indicating all critical points and all inflection points. What is the equation of the tangent line to the graph of $y = f(x)$ at the point $(0, 0)$?

3. [10 marks] A plane flies due north with constant speed 200 km/hr, at constant altitude 1 km above the ground. On the ground, a car travels due east along a straight highway, with constant speed 100 km/hr. At the moment the plane crosses over the highway, the car is 2 km east of the point on the road directly below the plane. How fast are the plane and the car separating one minute later?

University of Toronto
Faculty of Engineering
MAT 186H1F TERM TEST
FRIDAY, NOVEMBER 2, 2001, 4:10 PM
Duration: 50 minutes

Aids Allowed: Non-programmable calculator, to be supplied by student.

Instructions: Fill in the information on this page, and make sure this test contains 4 pages. Present your **solutions** in the space provided. Use the back of the preceding page if you need more space. The value for each question is indicated in square brackets beside each question number.

TOTAL MARKS: 40

1. [15 marks] Find the following limits:

   (a) [5 marks] $\lim_{x\to 0} \dfrac{\ln(1+x)-x}{x^2}$.

   (b) [4 marks] $\lim_{x\to 1} \left(\dfrac{x}{\ln x} - \dfrac{1}{x\ln x}\right)$.

   (c) [6 marks] $\lim_{x\to\infty} (x+e^x)^{2/x}$.

2. [15 marks] Let $f(x) = 5x^{2/3} - x^{5/3}$.

   (a) [6 marks] Find all critical points of f and determine if each critical point is a relative maximum point or minimum point.

   (b) [4 marks] On which intervals is the graph of f concave up? concave down?

   (c) [5 marks] Sketch the graph of $y = f(x)$, on the interval $-8 \le x \le 8$, indicating all critical points and all inflection points. What is the equation of the tangent line to the graph of $y = f(x)$ at the point $(0, 0)$?

3. [10 marks] At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr; ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 2 PM?

University of Toronto
Faculty of Engineering
MAT 186H1S TERM TEST
THURSDAY, MARCH 7, 2002
Duration: 90 minutes

Aids Allowed: Calculator, to be supplied by student.

Instructions: Present your **solutions** in the booklets provided.

TOTAL MARKS: 50 The value of each question is indicated in parantheses beside the question number.

1. [15 marks] The parts of this question are unrelated:

   (a) [5 marks] What is the equation of the tangent line to the function $f(x) = \dfrac{x^3 - 1}{\sqrt{x+1}}$ at the point $(x, y) = (0, -1)$?

   (b) [5 marks] Find $\dfrac{dy}{dx}$ at the point $(x, y) = (-1, 1)$ if $\tan(x + y) = x^5 + x^2 y$.

   (c) [5 marks] Use Newton's method to approximate a solution to the equation $x^3 + 4x - 2 = 0$ correct to 4 decimal places.

2. [15 marks] Let $f(x) = \dfrac{x^2 - x + 1}{x^2}$. Find the intervals on which the graph of $y = f(x)$ is increasing, decreasing, concave up, and concave down. Plot the graph of $y = f(x)$, labelling all critical points, all inflection points, and all asymptotes (if any.)

3. [10 marks] A light is at the top of a pole 20 m high, and a ball is dropped at the same height from a point 5 m from the light. How fast is the shadow of the ball moving along the ground 1 sec later? (The distance fallen by the ball t seconds after it has been dropped is given by $d = 4.9t^2$ m.)

4. [10 marks] A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is to be 10 m, find the dimensions of the window so that the greatest amount of light is admitted.