Friday, October 4, 2002, 11:10 AM
MAT 188H1F Linear Algebra Quiz 1
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. The value of each question is indicated in square brackets. **TOTAL MARKS: 20**

1. [5 marks] Find the angle (in radians or degrees) between the two vectors $\mathbf{u} = (1, 0, -2, 1)$ and $\mathbf{v} = (0, 1, 1, -1)$.

2. [5 marks] Find the equation in standard form of the plane containing the two lines with equations

$$\mathbf{x}(t) = (2, 1, 3) + t(0, -2, -1) \text{ and } \mathbf{y}(s) = (2, 1, 3) + s(1, 0, 2),$$

where s and t are parameters.

3. Consider the following system of equations:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ 2x_1 + 3x_2 + 3x_3 = 1 \\ -x_1 - 2x_2 - 2x_3 + x_4 = 0 \\ -x_2 - x_3 + 2x_4 = 1 \end{cases}$$

(a) [2 marks] Write down the augmented matrix for the above system.

(b) [5 marks] What is the row-reduced echelon form of the augmented matrix for the above system?

(c) [3 marks] What is the solution of the above system?

ANSWERS: 1. $135°$ 2. $4x + y - 2z = 3$

3.(a) $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 3 & 0 & 1 \\ -1 & -2 & -2 & 1 & 0 \\ 0 & -1 & -1 & 2 & 1 \end{array}\right)$ (b) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 2 \\ 0 & 1 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$ (c) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 - 3t \\ -1 - s + 2t \\ s \\ t \end{pmatrix}$,
where s and t are parameters.

Friday, October 4, 2002, 12:10 PM
MAT 188H1F Linear Algebra Quiz 1
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. The value of each question is indicated in square brackets. **TOTAL MARKS: 20**

1. [5 marks] Find the angle (in radians or degrees) between the two vectors $\mathbf{u} = (3, 1, 1, 2, 1)$ and $\mathbf{v} = (0, 2, 1, -2, 0)$.

2. [5 marks] Find the point of intersection of the line $\mathbf{x}(t) = (2, 1, 1) + t(-1, 0, 4)$ with the plane $x - 3y - z = 1$.

3. Consider the following system of equations:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ 2x_1 + 3x_2 + 3x_3 = 1 \\ -x_1 - 2x_2 - 2x_3 + x_4 = 0 \\ 3x_4 + x_2 + x_3 + 7x_4 = 5 \end{cases}$$

   (a) [2 marks] Write down the augmented matrix for the above system.

   (b) [5 marks] What is the row-reduced echelon form of the augmented matrix for the above system?

   (c) [3 marks] What is the solution of the above system?

ANSWERS: 1. $94.8°$, approximately. 2. $(x, y, z) = (13/5, 1, -7/5)$

3.(a) $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 3 & 0 & 1 \\ -1 & -2 & -2 & 1 & 0 \\ 3 & 1 & 1 & 7 & 5 \end{array}\right)$ (b) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 2 \\ 0 & 1 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$ (c) $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 - 3t \\ -1 - s + 2t \\ s \\ t \end{pmatrix}$,
where s and t are parameters.

Friday, October 18, 2002, 11:10 AM
MAT 188H1F Linear Algebra Quiz 2
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20**

1. [10 marks] Let $A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 0 & 2 \\ 2 & -4 & 3 \end{pmatrix}$. Find the following:

   (a) [5 marks] A^{-1}

   (b) [5 marks] the LU decomposition of A. DO NOT use any row interchanges.

2. [6 marks] Let $B = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$. Find the following:

   (a) [2 marks] $B^T B$

   (b) [2 marks] BB^T

   (c) [2 marks] the rank of BB^T

3. [4 marks] State whether the following statement is true or false, and explain why.

   If a linear system has no solutions, then the rank of the coefficient matrix must be less than the number of equations.

ANSWERS:

1.(a) $A^{-1} = \begin{pmatrix} -4/3 & 11/3 & -2/3 \\ -1/6 & 5/6 & -1/3 \\ 2/3 & -4/3 & 1/3 \end{pmatrix}$ (b) $L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 4 & 1 \end{pmatrix}$; $U = \begin{pmatrix} 1 & 2 & 4 \\ 0 & -2 & -2 \\ 0 & 0 & 3 \end{pmatrix}$

2.(a) (6) (b) $\begin{pmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 2 & -2 & 4 \end{pmatrix}$ (c) 1 3. True.

Friday, October 18, 2002, 12:10 PM
MAT 188H1F Linear Algebra Quiz 2
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20**

1. [10 marks] Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & -2 \\ 2 & 3 & -2 \end{pmatrix}$. Find the following:

   (a) [5 marks] A^{-1}

   (b) [5 marks] the LU decomposition of A. DO NOT use any row interchanges.

2. [6 marks] Let $B = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

   (a) [2 marks] Find B^2

   (b) [2 marks] Find B^3

   (c) [2 marks] Show that $(I - B)^{-1} = I + B + B^2$.

3. [4 marks] State whether the following statement is true or false, and explain why.

   A homogeneous linear system with two equations and four unknowns must have a two-parameter family of solutions.

ANSWERS:

1.(a) $A^{-1} = \begin{pmatrix} 6/5 & 7/5 & -4/5 \\ -2/5 & -4/5 & 3/5 \\ 3/5 & 1/5 & -2/5 \end{pmatrix}$ (b) $L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1/2 & 1 \end{pmatrix}$; $U = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -2 & -3 \\ 0 & 0 & -5/2 \end{pmatrix}$

2.(a) $B^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ (b) $B^3 = O$ (c) Check: $(I - B)(I + B + B^2) = I$

3. False.

Friday, November 1, 2002, 11:10 AM
MAT 188H1F Linear Algebra Quiz 3
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Each question is worth 5 marks. **TOTAL MARKS: 20**

1. Find $\det \begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & -1 & 2 \\ 1 & -1 & 0 & 5 \\ -1 & 2 & 1 & -2 \end{pmatrix}$.

2. Find the (classical) adjoint of the matrix $A = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & 1 \\ 1 & -1 & 5 \end{pmatrix}$

3. Use Cramer's rule to find the value of x_2 in the solution to the system of equations

$$\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 2x_1 + x_2 + x_3 = 0 \\ -x_1 + 2x_2 + 2x_3 = 3 \end{cases}$$

4. Suppose A is a 5×5 matrix such that $\det A = 3$. What then is the value of each of the following?

(a) $\det(A^T)$
(b) $\det(-2A)$
(c) $\det(E_{33}(4)A)$
(d) $\det(A^{-1})$
(e) $\det(\text{Adj}(A))$

ANSWERS: 1. -20 2. $\begin{pmatrix} 6 & -7 & -1 \\ 6 & 3 & -3 \\ 0 & 2 & 2 \end{pmatrix}$ 3. $x_2 = 14/15$

3.(a) 3 (b) -96 (c) 12 (d) $1/3$ (e) 81

Friday, November 1, 2002, 12:10 PM
MAT 188H1F Linear Algebra Quiz 3
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Each question is worth 5 marks. **TOTAL MARKS: 20**

1. Find $\det \begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 0 & -1 & 1 \\ 1 & -1 & 0 & 3 \\ -1 & 2 & 1 & -2 \end{pmatrix}$.

2. Find the (classical) adjoint of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 5 \end{pmatrix}$

3. Use Cramer's rule to find the value of x_1 in the solution to the system of equations

$$\begin{cases} x_1 & + & 2x_2 & - & x_3 & = & 1 \\ 2x_1 & + & x_2 & + & x_3 & = & 0 \\ -x_1 & + & 2x_2 & + & 2x_3 & = & 3 \end{cases}$$

4. Suppose A is a 5×5 matrix such that $\det A = -3$. What then is the value of each of the following?

(a) $\det(A^T)$

(b) $\det(-2A)$

(c) $\det(P_{23}A)$

(d) $\det(A^{-1})$

(e) $\det(\text{Adj}(A))$

ANSWERS: 1. -12 2. $\begin{pmatrix} 6 & -11 & 1 \\ 6 & 4 & -2 \\ 0 & 3 & 3 \end{pmatrix}$ 3. $x_1 = -9/15$

3.(a) -3 (b) 96 (c) 3 (d) $-1/3$ (e) 81

Friday, September 28, 2001, 9:10 AM
MAT 188H1F Quiz 1
Linear Algebra
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. Find the angle (in degrees or radians) between the two vectors

$$\mathbf{u} = (2, 3, -1) \text{ and } \mathbf{v} = (5, -4, 1).$$

2. Find the equation in standard form of the plane containing the three points

$$P(4, 2, 0), Q(0, 2, 1) \text{ and } R(-4, 0, 3).$$

3. Find a unit vector which is orthogonal to both $\mathbf{u} = (1, 0, -2, 1)$ and $\mathbf{v} = (0, 1, 1, -1)$.

4. Solve the following system of equations by using Gauss-Jordan elimination (i.e. use row operations on the augmented matrix to find its reduced row-echelon form):

$$x - y + 2z = -7$$
$$2x + y + z = 1$$
$$x + 3y - z = 10$$

Friday, September 28, 2001, 11:10 AM
MAT 188H1F Quiz 1
Linear Algebra
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. Find the angle (in degrees or radians) between the two vectors $\mathbf{u} = (5, 1, -1)$ and $\mathbf{v} = (-1, 2, 1)$.

2. Find the area of the triangle determined by the three points

$$P(2, 1, 3), Q(1, 0, 2) \text{ and } R(-1, 1, 2).$$

3. Find a unit vector which is orthogonal to both $\mathbf{u} = (1, -1, 0, 1)$ and $\mathbf{v} = (0, 2, 0, -4)$.

4. Solve the following system of equations by using Gauss-Jordan elimination (i.e. use row operations on the augmented matrix to find its reduced row-echelon form):

$$\begin{aligned} x + 3y + z &= 2 \\ -x + y + z &= -2 \\ 2x + y + 3z &= -3 \end{aligned}$$

Friday, October 12, 2001, 9:10 AM
MAT 188H1F Quiz 2
Linear Algebra
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (10 marks) Find the inverse of $A = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 2 \\ 2 & 4 & 3 \end{pmatrix}$ and use it to solve the system $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 30 \\ 60 \\ -30 \end{pmatrix}$ for x, y and z.

2. (6 marks) Let $A = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$. Find the following:

(a) the rank of A

(b) the rank of A^2

(c) the rank of A^3

3. (4 marks) State whether the following statement is true or false, and give a *brief* justification for your choice:

If a linear system has no solutions, then the rank of the coefficient matrix must be less than the number of equations.

Friday, October 12, 2001, 11:10 AM
MAT 188H1F Quiz 2
Linear Algebra
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (10 marks) Find the inverse of $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & 4 \\ 4 & 2 & 3 \end{pmatrix}$ and use it to solve the system $A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 30 \\ 60 \\ -30 \end{pmatrix}$ for x, y and z.

2. (6 marks) Let $A = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$. Find the following:

(a) the rank of A

(b) the rank of A^2

(c) the rank of A^3

3. (4 marks) State whether the following statement is true or false, and give a *brief* justification for your choice:

A linear system with more unknowns than equations always has infinitely many solutions.

Friday, October 26, 2001, 9:10 AM
MAT 188H1F Quiz 3
Linear Algebra
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. Find $\det \begin{pmatrix} 1 & 1 & 2 & -1 \\ 3 & 0 & 0 & 2 \\ 1 & -1 & 0 & 5 \\ -1 & 2 & 1 & -2 \end{pmatrix}$.

2. Find all values of c for which the matrix $A = \begin{pmatrix} 1 & c & 2 \\ 2 & 1 & 6 \\ 1 & -1 & c \end{pmatrix}$ is *not* invertible.

3. Use Cramer's rule to find the value of x_2 in the solution to the system of equations

$$\begin{cases} x_1 + 2x_2 - 2x_3 = 0 \\ 2x_1 + x_2 = 3 \\ -x_1 + x_2 + 2x_3 = 3 \end{cases}$$

4. Find the (classical) adjoint of the matrix $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & -1 & 5 \end{pmatrix}$

Friday, October 26, 2001, 11:10 AM
MAT 188H1F Quiz 3
Linear Algebra
Duration: 30 minutes
Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. Find $\det \begin{pmatrix} 1 & 1 & 2 & -1 \\ 1 & -1 & 0 & 5 \\ 3 & 0 & 0 & 2 \\ -1 & 2 & 1 & -2 \end{pmatrix}$.

2. Find all values of c for which the matrix $A = \begin{pmatrix} 1 & c & 2 \\ 2 & 2 & 6 \\ 1 & -2 & c \end{pmatrix}$ is *not* invertible.

3. Use Cramer's rule to find the value of x_1 in the solution to the system of equations

$$\begin{cases} x_1 + 2x_2 - 2x_3 = 0 \\ 2x_1 + x_2 = 3 \\ -x_1 + x_2 + 2x_3 = 3 \end{cases}$$

4. Find the (classical) adjoint of the matrix $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 5 \\ 2 & 1 & 3 \end{pmatrix}$