

Tuesday May 13, 2003, 3:30 PM
MAT 187H1F Calculua II Quiz 1

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20** Each question is worth 5 marks.

1. Find $\int \ln(x^2 + 1) dx$.

2. Find $\int \sin^5 x \cos^3 x dx$.

3. Find $\int \frac{\sqrt{x^2 - 16}}{x^2} dx$.

4. Find $\int \frac{1}{\sqrt{x^2 + 2x + 5}} dx$.

ANSWERS:

1. $x \ln(x^2 + 1) - 2x - 2 \tan^{-1} x + c$ 2. $\frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + c$

3. $\ln|x + \sqrt{x^2 - 16}| - \frac{\sqrt{x^2 - 16}}{x} + c$ 4. $\ln|\sqrt{x^2 + 2x + 5} + x + 1| + c$

Tuesday, May 20, 2003, 3:30 PM
MAT 187H1F Calculus II: Quiz 2

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20. Each question is worth 10 marks.**

1. Find $\int \frac{3x^3 + 5x^2 + 5x + 1}{(x + 1)^2(x^2 + 1)} dx$

2. Consider the curve with parametric equations

$$x = 2t^2 \text{ and } y = \frac{t^3}{3} - 4t.$$

(a) (5 marks) Find the values of both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $t = 1$.

(b) (5 marks) Find the length of the curve on the interval $0 \leq t \leq 3$.

ANSWERS:

1. $\ln|x + 1| + (1 + x)^{-1} + \ln(x^2 + 1) + \text{Tan}^{-1}x + c$

2.(a) $\frac{dy}{dx} = -\frac{3}{4}$; $\frac{d^2y}{dx^2} = \frac{5}{16}$ (b) 21

Tuesday, May 27, 2003, 3:30 PM
MAT 187H1F Calculus II, Quiz 3

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20.** Each question is worth 5 marks.

1. Plot the two polar graphs with polar equations $r = 2$ and $r^2 = 8 \cos(2\theta)$.
2. Find the area of the region within one loop (or leaf) of the polar graph with equation $r^2 = 8 \cos(2\theta)$.
3. What is the area of the region outside the circle with polar equation $r = 2$ but inside the graph with polar equation $r^2 = 8 \cos(2\theta)$?
4. What is the length of the logarithmic spiral $r = e^\theta$ for $\pi \leq \theta \leq 2\pi$?

ANSWERS:

2. 4 3. $4\sqrt{3} - \frac{4}{3}\pi$ 4. $\sqrt{2}e^\pi(e^\pi - 1)$

Tuesday, June 10, 2003, 3:30 PM
MAT 187H1F Calculus II Quiz 4

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20.**

1. (6 marks) Use power series that you already know to find the first three non-zero terms in the Maclaurin series for each of the following functions:

(a) (3 marks) $f(x) = x^2 e^{-x}$

(b) (3 marks) $g(x) = (1 + 4x^2)^{-1/3}$

2. (4 marks) Find the radius of convergence of the power series $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n} x^n$

3. (5 marks) Use the Maclaurin series of $\sin(x^2)$ to approximate the integral $\int_0^{0.5} \sin(x^2) dx$ correct to four decimal places.

4. (5 marks) Find the sum of the power series $\sum_{n=0}^{\infty} \frac{1}{n+1} x^{(n+2)}$; and its open interval of convergence.

ANSWERS:

1.(a) $x^2 - x^3 + (1/2)x^4$ (b) $1 - \frac{4}{3}x^2 + \frac{32}{15}x^4$ 2. 1 3. 0.041480654...

4. $-x \ln(1 - x)$

Tuesday, June 17, 2003, 3:30 PM
MAT 187H1F Calculus II Last Quiz

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20.**

1. (6 marks) For each of the following two series, find its sum or explain why it diverges:

(a) (3 marks) $\sum_{n=0}^{\infty} \left(\frac{-2}{3}\right)^n$.

(b) (3 marks) $\sum_{n=1}^{\infty} \text{Tan}^{-1}n$.

2. (4 marks) Use the integral test to determine if the infinite series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$ converges or diverges.

3. (5 marks) Determine if the series $\sum_{n=0}^{\infty} \frac{n^3}{3^n}$ converges or diverges.

4. (5 marks) Find $\frac{\partial^2 z}{\partial y \partial x}$ if $z = \text{Tan}^{-1}(y/x)$.

ANSWERS:

- 1(a) 3/5 (b) Diverges by n th term test. 2. Converges.

3. Converges by ratio test. 4. $\frac{y^2 - x^2}{(x^2 + y^2)^2}$

Wednesday, January 22, 2003, 5 PM

MAT 187H1S Calculus II, Quiz 1

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Each question is worth 5 marks. **TOTAL MARKS: 20**

1. Find $\int \sec^4 x \tan x \, dx$.

2. Find $\int x^2 \sqrt{5 - x^3} \, dx$.

3. Find $\int \frac{1}{x^2 \sqrt{1 - x^2}} \, dx$.

4. Find $\int \text{Tan}^{-1} x \, dx$.

ANSWERS:

1. $\frac{1}{4} \sec^4 x + c$ 2. $-\frac{2}{9}(5 - x^3)^{3/2} + c$

3. $-\frac{\sqrt{1 - x^2}}{x} + c$ 4. $x \text{Tan}^{-1} x - \ln \sqrt{1 + x^2} + c$

Friday, January 24, 2003, 3 PM
MAT 187H1S Calculus II, Quiz 1

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Each question is worth 5 marks. **TOTAL MARKS: 20**

1. Find $\int \sin^2 x \cos^3 x \, dx$.

2. Find $\int x^2 \sqrt{5 + x^3} \, dx$.

3. Find $\int \frac{\sqrt{x^2 - 4}}{x^2} \, dx$.

4. Find $\int x \ln x \, dx$.

ANSWERS:

1. $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$ 2. $\frac{2}{9}(5 + x^3)^{3/2} + c$

3. $\ln|x + \sqrt{x^2 - 4}| - \frac{\sqrt{x^2 - 4}}{x} + c$ 4. $\frac{x^2}{2} \ln x - \frac{1}{4}x^2 + c$

Wednesday, February 5, 5:10 PM, 2003

MAT 187H1S Calculus II, Quiz 2

Duration: 30 minutes

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20**

1. (10 marks) Find $\int \frac{9x^2 - 5x + 14}{x^4 - 3x^2 - 4} dx$

2. (10 marks) Find $\int \frac{x}{(5 + 4x - x^2)^{3/2}} dx$

ANSWERS:

1. $2 \ln |x - 2| - 3 \ln |x + 2| + \ln \sqrt{x^2 + 1} - \tan^{-1} x + c$

2. $\frac{1}{9} \frac{2x + 5}{\sqrt{5 + 4x - x^2}} + c$

Friday, February 7, 3:10 PM, 2003

MAT 187H1S Calculus II, Quiz 2

Duration: 30 minutes

Only aids permitted: a Casio 260, Sharp 520, or Texas Instrument 30 calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20**

1. (10 marks) Find $\int \frac{x^3 - 7x^2 + 5x - 1}{x^4 - 2x^3 + 2x^2 - 2x + 1} dx$

2. (10 marks) Find $\int \frac{x}{(5 + 4x + x^2)^2} dx$

ANSWERS:

1. $-2 \ln|x - 1| + (x - 1)^{-1} + 3 \ln \sqrt{x^2 + 1} - 2 \text{Tan}^{-1} x + c$

2. $\frac{1}{2} \frac{x^2 + 2x}{5 + 4x + x^2} - \text{Tan}^{-1}(x + 2) + c$

Wednesday 5 PM, March 26, 2003

MAT 187H1S Calculus II Quiz 3

Duration: 30 minutes

Only aids permitted: a Casio 260, Sharp 520, or
Texas Instrument 30 calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20. Each question is worth 5 marks.**

1. Use the first four non-zero terms of the Maclaurin series for e^{-x^2} to approximate

$$\int_0^1 e^{-x^2} dx.$$

2. Use appropriate series — *not* L'Hôpital's Rule — to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - \frac{1}{2}x}{x^2}.$$

3. Find the open interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} x^n$

4. Find the sum of the power series $\sum_{n=1}^{\infty} \frac{x^{n+1}}{n}$, if $|x| < 1$.

ANSWERS:

1. $26/35 = 0.742857142 \dots$ 2. $-1/8$ 3. $-\frac{1}{2} < x < \frac{1}{2}$ 4. $-x \ln(1-x)$

Friday 3 PM, March 28, 2003
MAT 187H1S Calculus II Quiz 3

Duration: 30 minutes

Only aids permitted: a Casio 260, Sharp 520, or
Texas Instrument 30 calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. **TOTAL MARKS: 20. Each question is worth 5 marks.**

1. Use the first four non-zero terms of the Maclaurin series for $\sin x^2$ to approximate

$$\int_0^1 \sin x^2 dx.$$

2. Use appropriate series — *not* L'Hôpital's Rule — to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{1 + \frac{1}{2}x - \sqrt{1+x}}{x^2}.$$

3. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} x^{2n}$

4. Find the sum of the power series $\sum_{n=1}^{\infty} nx^{n+1}$, if $|x| < 1$.

ANSWERS:

1. 0.310268... 2. 1/8 3. $\frac{1}{2}$ 4. $\frac{x^2}{(1-x)^2}$

Tuesday May 14, 2002, 3:30 PM

MAT 187H1F Quiz 1

Calculus II

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (10 marks) Let $f(x) = \text{Sec}^{-1}x + \text{Cot}^{-1}\sqrt{x^2 - 1}$.

(a) (7 marks) Find $f'(x)$ in simplified form. Is $f(x)$ a constant function?

(b) (3 marks) What is the constant value of $f(x)$ for $x \leq -1$?

2. (4 marks; 2 marks each) Find the exact value of

(a) $\sin(\text{Sin}^{-1}(1/3))$

(b) $\text{Sin}^{-1}(\sin(7\pi/4))$

3. (6 marks) Find $\int \frac{1}{\sqrt{x}(1+x^{1/4})} dx$.

Tuesday, May 21, 2002, 3:30 PM

MAT 187H1F Quiz 2

Calculus II

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (5 marks) Find $\int \sin^{-1} x \, dx$
2. (5 marks) Suppose the amount of a drug injected into the body decreases at a rate proportional to the amount still present. If a dose decreases by 6% in the first hour, when will it decrease to one-half its original amount?
3. (5 marks) Find $\int \sin^3 x \cos^4 x \, dx$
4. (5 marks) Find $\int \frac{\sqrt{x^2 - 1}}{x^2} \, dx$

Tuesday, May 28, 2002, 3:30 PM

MAT 187H1F Quiz 3

Calculus II

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (10 marks) Find $\int \frac{1}{\sqrt{x^2 + 4x + 13}} dx$

2. (10 marks) Find $\int \frac{3x^3 - 2x^2 - 7}{x^4 + x^2 - 2} dx$

Tuesday, June 11, 2002, 3:30 PM

MAT 187H1F Quiz 4

Calculus II

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (7 marks) Let $\mathbf{u} = 3t\mathbf{i} + \cos t\mathbf{j} - e^t\mathbf{k}$; let $\mathbf{v} = t^2\mathbf{i} - 6\mathbf{k}$. Simplify

$$\frac{d\mathbf{u}}{dt} \times \mathbf{v} + \int \mathbf{v} dt.$$

2. (5 marks) Find the length of the curve with parametric equations

$$x = 2t^{3/2}; y = 3t; z = -\sqrt{7}t$$

for $0 \leq t \leq 1$.

3. (8 marks) A particle is moving through space under the influence of an acceleration given by $\mathbf{a} = 2t\mathbf{j} - 4\mathbf{k}$. Suppose the particle starts ($t = 0$) at the origin $(x, y, z) = (0, 0, 0)$ with initial velocity $\mathbf{v}_0 = 4\mathbf{i} - \mathbf{j} + 4\mathbf{k}$. Find where and when the particle passes through the xy -plane again, and with what speed. (Recall: the xy -plane has equation $z = 0$.)

Tuesday, June 18, 2002, 3:30 PM

MAT 187H1F Last Quiz

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20. Each Questions is worth 5 marks.

1. Find the first three non-zero terms of the Maclaurin series of $f(x) = x^2 \sin x$.

2. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$

3. Find the sum of the series $\sum_{n=0}^{\infty} (n+1)x^n$

4. Approximate the value of $\int_0^{0.5} \frac{1}{(1+x^2)^{1/3}} dx$ by integrating the first four non-zero terms of the Maclaurin series of $(1+x^2)^{-1/3}$.

Tuesday 5 PM, January 23, 2001

MAT 187H1S Quiz 1

Calculus II

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (10 marks) Let $f(x) = \sin^{-1}\left(\frac{2x}{x^2 + 1}\right)$. Find

(a) (4 marks) $f(-\sqrt{3})$

(b) (6 marks) $f'(-\sqrt{3})$

2. (5 marks) Find $\int \frac{x + 2}{\sqrt{x + 5}} dx$.

3. (5 marks) If one-half of a sample of radioactive substance decays in 15 days, then how long does it take for 90% of the sample to decay?

Wednesday 5 PM, January 24, 2001

MAT 187H1S Quiz 1

Calculus II

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (10 marks) Let $f(x) = \text{Sec}^{-1}(-\sqrt{x^2 + 1})$. Find

(a) (4 marks) $f(-1)$

(b) (6 marks) $f'(-1)$

2. (5 marks) Find $\int \frac{x + 7}{\sqrt{x + 1}} dx$.

3. (5 marks) If 10% of a sample of radioactive material decays in 3 seconds, what is its half-life?

Tuesday, February 5, 11:10 AM, 2002

MAT 187H1S Quiz 2

Calculus II

Duration: 30 minutes

Only aids permitted: a Casio 260, Sharp 520, or
Texas Instrument 30 calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this page.

TOTAL MARKS: 20

1. (5 marks) Find $\int \sin^4 x \cos^3 x dx$

2. (5 marks) Find $\int x \tan^{-1} x dx$

3. (10 marks) Find $\int \frac{2x - 3}{\sqrt{x^2 + 6x + 13}} dx$

ANSWERS: 1. $\frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c$

2. $\frac{x^2 + 1}{2} \tan^{-1} x - \frac{1}{2} x + c$

3. $2\sqrt{x^2 + 6x + 13} - 9 \ln |\sqrt{x^2 + 6x + 13} + x + 3| + c$

Wednesday 5 PM, February 6, 2002

MAT 187H1S Quiz 2

Calculus II

Duration: 30 minutes

Only aids permitted: a Casio 260, Sharp 520, or
Texas Instrument 30 calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this page.

TOTAL MARKS: 20

1. (5 marks) Find $\int \tan^5 x \sec^3 x \, dx$
2. (5 marks) Find $\int \sin^{-1} x \, dx$
3. (10 marks) Find $\int \frac{2x - 1}{(4x - x^2)^{3/2}} \, dx$

ANSWERS: 1. $\frac{1}{3} \sec^3 x + \frac{2}{5} \sec^5 x + \frac{1}{7} \sec^7 x + c$

2. $x \sin^{-1} x + \sqrt{1 - x^2} + c$

3. $\frac{1}{4} \frac{2 + 3x}{\sqrt{4x - x^2}} + c$

Tuesday 11 AM, March 26, 2002

MAT 187H1S Quiz 3

Calculus II

Duration: 30 minutes

Only aids permitted: a Casio 260, Sharp 520, or
Texas Instrument 30 calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (10 marks) For each of the following functions, write down the first three non-zero, simplified terms of its Maclaurin series:

(a) $\sin(2x)$

(b) $\frac{x}{1+x}$

(c) $(1-x^2)^{-3/2}$

(d) $\frac{1}{x^2+5x+6}$

2. (5 marks) Find the open interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n$

3. (5 marks) Find the sum of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n+1}$, if $|x| < 1$.

ANSWERS: 1.(a) $2x - \frac{4}{3}x^3 + \frac{4}{15}x^5$ (b) $x - x^2 + x^3$

1.(c) $1 + \frac{3}{2}x^2 + \frac{15}{8}x^4$ (d) $\frac{1}{6} - \frac{5}{36}x + \frac{19}{216}x^2$

2. $-4 < x < 4$ 3. $-\frac{\ln(1-x)}{x}$

Wednesday 5 PM, March 27, 2002

MAT 187H1S Quiz 3

Calculus II

Duration: 30 minutes

Only aids permitted: a Casio 260, Sharp 520, or
Texas Instrument 30 calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (10 marks) For each of the following functions, write down the first three non-zero, simplified terms of its Maclaurin series:

(a) e^{-x}

(b) $\frac{x^2}{1-x}$

(c) $\frac{1}{\sqrt{4+x^2}}$

(d) $\frac{1}{x^2+5x+6}$

2. (5 marks) Find the open interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-e)^n}{n^2} x^{2n}$

3. (5 marks) Find the sum of the power series $\sum_{n=1}^{\infty} (n+1)x^{n-1}$, if $|x| < 1$.

ANSWERS: 1.(a) $1 - x + \frac{1}{2}x^2$ (b) $x^2 + x^3 + x^4$

1.(c) $\frac{1}{2} - \frac{1}{16}x^2 + \frac{3}{256}x^4$ (d) $\frac{1}{6} - \frac{5}{36}x + \frac{19}{216}x^2$

2. $-\frac{1}{\sqrt{e}} < x < \frac{1}{\sqrt{e}}$ 3. $\frac{2-x}{(1-x)^2}$