

NAME: _____ STU. NO: _____

Monday, September 30, 2002, 9:10 AM
MAT 186H1F Calculus I Quiz 1

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.
Do not use L'Hopital's Rule for this quiz.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. Find the following limit or explain why it does not exist:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}.$$

2. Find the equations for the tangent and normal lines to the curve $f(x) = \sqrt{x} + 5$ at the point $(x, y) = (4, 7)$.

3. Find the following limit or explain why it does not exist:

$$\lim_{x \rightarrow 4} \frac{|x-4|}{x^2 - 3x - 4}.$$

4. Express the function $f(x) = \begin{cases} 1+x^2, & x < -1 \\ x-4, & -1 < x < 3 \\ 1-x^2, & x > 3 \end{cases}$ in terms of Heaviside functions.

ANSWERS: 1. 1 2. Tangent: $4y = x + 24$; Normal: $y + 4x = 23$.

3. Limit does not exist; the one-sided limits are not the same.

4. $f(x) = (1+x^2)(1-H(x+1)) + (x-4)(H(x+1) - H(x-3)) + (1-x^2)H(x-3)$

NAME: _____ STU. NO: _____

Wednesday, October 2, 2002, 4:10 PM
MAT 186H1F Calculus I Quiz 1

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.
Do not use L'Hopital's Rule for this quiz.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. Find the equations for the tangent and normal lines to the curve $f(x) = \sqrt{x} + 3$ at the point $(x, y) = (4, 5)$.
2. Find the following limit or explain why it does not exist:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x).$$

3. Find the following limit or explain why it does not exist:

$$\lim_{x \rightarrow 5} \frac{|x^2 - 25|}{x^2 - 25}.$$

4. Express the function $f(x) = \begin{cases} x^2 - 3, & x < 1 \\ x, & 1 < x < 4 \\ -x^2, & x > 4 \end{cases}$ in terms of Heaviside functions.

ANSWERS: 1. Tangent: $4y = x + 16$; Normal: $y + 4x = 21$. 2. $1/2$

3. Limit does not exist; one-sided limits are different.

4. $f(x) = (x^2 - 3)(1 - H(x - 1)) + x(H(x - 1) - H(x - 4)) - x^2H(x - 4)$

NAME: _____ STU. NO: _____

Monday, October 21, 2002, 9:10 AM
MAT 186H1F: Calculus I Quiz 2

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. The position on the x -axis of a particle at time t is given by

$$x(t) = t^3 - 9t^2 + 15t - 2.$$

Find the velocity, the speed and the acceleration of the particle at time $t = 2$.

2. Find $f'(x)$ if $f(x) = xe^{\tan(\ln x)}$.

3. Find $\frac{dy}{dx}$ at the point $(x, y) = (0, 0)$ if $\sin(x + y) = 1 - \sqrt{1 + y}$.

4. Use Newton's method to approximate a solution to the equation $x^5 - x - 2 = 0$: start with $x_1 = 1.5$; calculate x_2 and x_3 .

ANSWERS: 1. At $t = 2$, $v = -9$, $a = -6$, and the speed is $|v| = 9$.

2. $f'(x) = e^{\tan(\ln x)} + \sec^2(\ln x) e^{\tan(\ln x)}$

3. $y' = -2/3$ 4. $x_2 = 1.33162\dots$; $x_3 = 1.27352\dots$

NAME: _____ STU. NO: _____

Wednesday, October 23, 2002, 4:10 PM
MAT 186H1F: Calculus I Quiz 2

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. The position on the x -axis of a particle at time t is given by

$$x(t) = t^3 - 9t^2 + 15t - 2.$$

Find the velocity, the speed and the acceleration of the particle at time $t = 3$.

2. Find $f'(x)$ if $f(x) = x \sec(\ln(1 + e^x))$.
3. Find $\frac{dy}{dx}$ at the point $(x, y) = (0, 0)$ if $\tan(2x + y) = e^x - \sqrt{1 + y}$.
4. Use Newton's method to approximate a solution to the equation $x^4 + x - 3 = 0$: start with $x_1 = 1.5$; calculate x_2 and x_3 .

ANSWERS: 1. At $t = 3$, $v = -12$, $a = 0$, and the speed is $|v| = 12$.

2. $f'(x) = \sec(\ln(1 + e^x)) + x \sec(\ln(1 + e^x)) \tan(\ln(1 + e^x)) \frac{e^x}{1 + e^x}$

3. $y' = -2/3$ 4. $x_2 = 1.25431\dots$; $x_3 = 1.17228\dots$

NAME: _____ STU. NO: _____

Monday, November 18, 2002, 9:10 AM
MAT 186H1F Calculus I Quiz 3

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth five marks.

TOTAL MARKS: 20

1. Find the average value of the function $f(x) = \cos x$, on the interval $0 \leq x \leq \pi/2$.
2. Find $\int_1^4 \frac{1}{x^{3/2}\sqrt{5-x}} dx$. (Hint: start by letting $u = \frac{1}{\sqrt{x}}$.)
3. If $F(x) = \int_{\cos x}^{\sin x} \frac{1}{\sqrt{2t^2+1}} dt$, what is the value of $F'(\pi/4)$?
4. If 10% of a sample of radioactive material decays in 5 min, how long will it take for 75% of the sample to decay?

ANSWERS: 1. $2/\pi$ 2. $3/5$ 3. 1 4. 65.8 min

NAME: _____ STU. NO: _____

Wednesday, November 20, 2002, 4:10 PM

MAT 186H1F Calculus I Quiz 3

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth five marks.

TOTAL MARKS: 20

1. Find the average value of the function $f(x) = \sec^2 x$, on the interval $0 \leq x \leq \pi/4$.
2. Find $\int_1^4 \frac{1}{x^{3/2}\sqrt{5+x}} dx$. (Hint: start by letting $u = \frac{1}{\sqrt{x}}$.)
3. If $F(x) = \int_{\sin x}^{\cos x} \frac{1}{\sqrt{2t^2+1}} dt$, what is the value of $F'(\pi/4)$?
4. The y -intercept of the normal line to a curve at any point is always equal to 4. If the curve passes through the point $(2, 3)$, find its equation.

ANSWERS: 1. $4/\pi$ 2. $(2\sqrt{6}-3)/5$ 3. 1 4. $y^2 - 8y + x^2 + 11 = 0$.

Monday, October 1, 2001, 9 AM

MAT 186H1F Quiz 1

Calculus I

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Do not use L'Hopital's Rule for this quiz.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. Find the following limit or explain why it does not exist:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 9x} - x).$$

2. What is the equation of the tangent line to the graph of $f(x) = \frac{x^2 - 1}{x^2 + 1}$ at the point $(x, y) = (3, \frac{4}{5})$?

3. Find the following limit or explain why it does not exist:

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{|x - 1|}.$$

4. Sketch the graph of $y = x^2(1 - H(x+1)) + xH(x-2)$, where $H(x)$ is the Heaviside function.

Friday, October 5, 2001

MAT 186H1F Quiz 1

Calculus I

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Do not use L'Hopital's Rule for this quiz.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. Find the equation of the tangent line to the graph of $f(x) = \frac{x^3 + 2x}{x^2 + 1}$ at the point $(x, y) = (1, \frac{3}{2})$

2. Find the following limit or explain why it does not exist:

$$\lim_{x \rightarrow -\infty} (\sqrt{x^2 + 5x} + x)$$

3. Find the following limit or explain why it does not exist:

$$\lim_{x \rightarrow 3} \frac{x^2 + x - 6}{x^2 - 9}$$

4. Plot the graph of $f(x) = x^2H(x) + (3 - x)H(x + 3)$, where $H(x)$ is the Heaviside function.

Monday, October 15, 2001, 9:10 AM

MAT 186H1F Quiz 2

Calculus I

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. The position on the x -axis of a particle at time t is given by

$$x(t) = t^3 - 9t^2 + 15t - 2.$$

During which times is the particle moving to the left?

2. Find $f''(x)$ if $f(x) = x^2e^{-3x}$.
3. Find $\frac{dy}{dx}$ at the point $(x, y) = (0, 0)$ if $\tan y = 2x + xy$.
4. Find $\frac{dy}{dx}$ at the point $(x, y) = (\pi/2, 1)$ if $y = (\sin x)^{\sqrt{x}}$.

Friday, October 19, 2001, 4:10 PM

MAT 186H1F Quiz 2

Calculus I

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet. Each question is worth 5 marks.

TOTAL MARKS: 20

1. For which values of x is the function $f(x) = x^2 e^{-x}$ increasing?
2. Find the value of $\frac{dy}{dx}$ at the point $(x, y) = \left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ if $x \cos y - y \sin x = 0$.
3. Use Newton's method to approximate a solution to the equation

$$x^3 - x^2 + x - 2 = 0 :$$

start with $x_1 = 1.5$ and calculate x_2 and x_3 .

4. Find the value of $\frac{dy}{dx}$ at the point $(x, y) = (2, 1)$ if $y = \left(\frac{2}{x}\right)^{3/x}$.

Monday, November 19, 2001, 9:10 AM

MAT 186H1F Quiz 3

Calculus I

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (5 marks) Find $\int \frac{x^{5/2} + x^2 + 1}{x} dx$.
2. (5 marks) Find $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$.
3. (5 marks) Find $\lim_{n \rightarrow \infty} \left(1 - \frac{2}{n}\right)^n$.
4. (5 marks) The y -intercept of the normal line to a curve at any point is always equal to 2. If the curve passes through the point $(3, 4)$ find its equation.

Friday, November 23, 2001, 4:10 PM

MAT 186H1F Quiz 3

Calculus I

Duration: 30 minutes

Only aids allowed for this quiz: a non-programmable calculator.

Instructions: Present your solutions to the following questions on this sheet, using both sides. Make sure to fill in your name and student number at the top of this sheet.

TOTAL MARKS: 20

1. (5 marks) Find $\int_1^4 \frac{x^3 + x + 1}{x^2} dx$.
2. (5 marks) Find $F'(2)$ if $F(x) = \int_1^{\sqrt{x}} \frac{\sqrt{t^2 + 2}}{t^2 + 7} dt$.
3. (5 marks) When the brakes of a car are applied, they produce a constant deceleration of 10 m/s^2 . What is the distance, from the point of application of the brakes, required to stop a car travelling at 40 m/sec ?
4. (5 marks) The y -intercept of the normal line to a curve at any point is always equal to 5. If the curve passes through the point $(3, 4)$ find its equation.