University of Toronto

Foundational Concepts in Calculus and Algebra

Friday, August 24, 2007

Duration: 90 minutes

Instructions: Answer all questions. Present your solutions in the space provided. The value for each question is indicated in parantheses beside the question number.

TOTAL MARKS: 100

NAME: _____

STUDENT NUMBER: _____

SIGNATURE: _____

LOCATION: (eg BA 3004) _____

MARKER'S REPORT:

QUESTION	MARK
Q 1	
Q 2	
Q 3	
Q 4	
Q 5	
Q 6	
Q 7	
Q 8	
Q 9	
TOTAL	

1. [20marks] Decide if the following statements are true or false. Circle either T for true, or F for false, at the right of each question.

(a) $(-1)^{1/3} = -1$ T F

(b) $\sqrt{a^2} = a$ T F

(c) $\sqrt{a^2 + b^2} = a + b$ T F

(d) $\sqrt{e^{2x} + 2 + e^{-2x}} = e^x + e^{-x}$ T F

(e) $\dfrac{1}{a+b} = \dfrac{1}{a} + \dfrac{1}{b}$, if $a \neq 0, b \neq 0, a + b \neq 0$ T F

(f) $\dfrac{a+b}{\frac{1}{a} + \frac{1}{b}} = ab$, if $a \neq 0, b \neq 0, a + b \neq 0$ T F

(g) $\sin(\alpha + \beta) = \sin\alpha + \sin\beta$ T F

(h) $\cos(2\theta) = 2\cos\theta$ T F

(i) $|x + 2| \leq 3 \Leftrightarrow -5 \leq x \leq 1$ T F

(j) $|x|^2 = x^2$ T F

(k) $|x|^3 = x^3$ T F

(l) $\ln(e^x) = x$ T F

(m) $e^a e^b = (e^a)^b$ T F

(n) $\ln(MN) = \ln M + \ln N$, if $M > 0, N > 0$. T F

(o) $\ln\left(\dfrac{1}{x^2+1}\right) = -\ln(x^2 + 1)$ T F

(p) $\ln\left(\dfrac{M}{N}\right) = \dfrac{\ln M}{\ln N}$, if $M > 0, N > 0$. T F

(q) The graph of $y = \ln(x^2 + 1)$ is always increasing. T F

(r) There is only one real solution to the equation $x^3 = 1 - x$. T F

(s) There are three real solutions to the equation $e^x = 3x^2$. T F

(t) $\sqrt{x^2 + x} - x = \dfrac{1}{1 + \sqrt{1 + 1/x}}$, if $x > 0$. T F

2. [20 marks; 4 marks for each part] Solve for x if:

(a) $x^2 + 4x - 5 = 0$.

(b) $x^3 + 3x^2 - x - 3 = 0$

(c) $\dfrac{(x-3)(x+2)}{x} \geq 0$

(d) $|2x - 4| \leq |x + 3|$

(e) $\log_3 x + \log_3(x - 6) = 3$

3. [12 marks; 4 marks for each part] Suppose $\sin\theta = \frac{2}{5}$ and $\cos\theta < 0$. Find the exact values of each of the following:

(a) $\cos\theta$

(b) $\cos(2\theta)$

(c) $\sin\left(\frac{\theta}{2}\right)$

4. [10 marks] Find the values of A, B, C and D, such that

$$\frac{4x+3}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}.$$

5. [8 marks; 4 marks for each part.] Find the following:

(a) the vertex of the parabola with equation $y = 5 + 6x - x^2$.

(b) the centre and radius of the circle with equation $x^2 + 2x + y^2 - 4y = 4$.

6. [8 marks; 4 marks for each part] Sketch the graphs of the following functions, labelling any asymptotes, if any.

   (a) $f(x) = -e^{-3x}$

   (b) $g(x) = \dfrac{x^2 + 1}{x + 1}$

7. [8 marks] Suppose the velocity of a particle at time t is given by $v = t^2 - 4t - 5$.

(a) [5 marks] What is the *speed* of the particle at time t? Sketch its graph.

(b) [3 marks] What is the maximum speed of the particle for $0 \leq t \leq 5$?

8. [8 marks] Let $f(x) = \dfrac{2x+3}{x-5}$. Find the formula for $f^{-1}(x)$, and sketch the graphs of both f and f^{-1} on the same graph.

9.[6 marks] Suppose a bacterial colony is growing exponentially so that its mass doubles every 20 minutes. How long will it take until the colony triples in mass?