

## FIRST YEAR MATH: General Advice

July 6, 2010

**Written Tests:** All tests and exams require you to show your work and explain what you are doing. The final answer, even if it is correct, will be worth very little unless you have *explained* your solution. Moreover, final answers should always be simplified.

It is not our responsibility to figure out what you have done; you are supposed to make it clear what you are doing.

**Homework:** Lectures will expose you to new ideas and show you what kind of examples you should be able to do. But none of the material from lectures will really sink in until you try the homework. *To be successful in a math course you must do the homework.* You can get help with homework from your TA, or from your lecturer during his or her office hours, or you can drop by the Math Aid Office, GB 149. Also, there is usually help provided by the publishers on web sites associated with their textbooks.

It is advisable to do your homework on a regular (weekly) basis. Leaving things until the night before a quiz or a test is asking for trouble.

**Notation:** Mathematics has its own set of symbols. If you use them, you must use them correctly. (You will lose marks on tests or exams if your notation is incorrect.) Here are some examples of common errors:

**Example 1:** Suppose you are differentiating the function  $y = x^2$ . If you write

$$\begin{aligned}y &= x^2 \\ &= 2x\end{aligned}$$

it is incorrect, even though most people would know what you are doing. Of course, you should write

$$\begin{aligned}y &= x^2 \\ \Rightarrow y' &= 2x\end{aligned}$$

Or you could simply write

$$\frac{dx^2}{dx} = 2x.$$

**Example 2:** Don't confuse implication ( $\Rightarrow$ ) with equality ( $=$ ). The symbol  $\Rightarrow$  is a logical connective and means "If ... then ...". For example,

$$\text{if } y = x^2, \text{ then } y' = 2x,$$

can be written as

$$y = x^2 \Rightarrow y' = 2x.$$

But writing something like

$$\begin{aligned}ax^2 + bx + c &\Rightarrow 0 \\ x &\Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\end{aligned}$$

is abuse of notation.

**Example 3:** The following expressions

$$\frac{0}{0}, \frac{\infty}{\infty}, 0^0, 0 \cdot \infty, 1^\infty, \infty^0 \text{ and } \infty - \infty$$

are indeterminate. When evaluating limits of this type, which may or may not exist, don't *equate* the limit to one of the above expressions. For example, writing

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} = \frac{0}{0}$$

is incorrect; the limit is actually equal to  $\frac{3}{2}$ . You *can* say the limit is of the form  $\frac{0}{0}$ , or that when  $x = 1$ ,

$$\frac{x^3 - 1}{x^2 - 1} = \frac{1 - 1}{1 - 1} = \frac{0}{0},$$

which is indeterminate.

**Example 4:** When reducing a matrix, don't put equal ( = ) signs between matrices which *aren't* equal:

$$\begin{pmatrix} 1 & 3 & -2 \\ -2 & 3 & 1 \\ 1 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & -2 \\ 0 & 9 & -3 \\ 0 & -1 & 1 \end{pmatrix}$$

is incorrect. Instead, use an arrow (  $\rightarrow$  ) between row equivalent matrices:

$$\begin{pmatrix} 1 & 3 & -2 \\ -2 & 3 & 1 \\ 1 & 2 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & -2 \\ 0 & 9 & -3 \\ 0 & -1 & 1 \end{pmatrix}.$$

**Example 5:** An indefinite integral always includes an arbitrary constant. To write

$$\int x^2 dx = \frac{x^3}{3}$$

is incomplete, and will cost you a mark everytime. Correct is:

$$\int x^2 dx = \frac{x^3}{3} + c.$$

**Example 6:** The correct indefinite integral of  $\frac{1}{x}$  with respect to  $x$  is:

$$\int \frac{1}{x} dx = \ln |x| + c.$$

Without the absolute value it is wrong, and will cost you a mark everytime. Some students even write

$$\int \frac{1}{x} dx = \ln x,$$

which is doubly wrong – and will cost you *two* marks.

**Logic:** Errors in logic that show up in your written solutions will cost you marks on a test or exam, and may even make your whole “solution” worthless – even if your calculations are correct. Here are some examples:

**Example 7:** The converse of a true statement is *not* necessarily a true statement. For example, it is true that if the infinite series

$$\sum_{n=1}^{\infty} a_n$$

converges, then

$$\lim_{n \rightarrow \infty} a_n = 0.$$

But it is not necessarily true that if

$$\lim_{n \rightarrow \infty} a_n = 0,$$

then the infinite series

$$\sum_{n=1}^{\infty} a_n$$

converges. For example, the harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges, even though

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0.$$

**Example 8:** To show a statement is *false* you only have to exhibit one counterexample. For example, the statement

If the number of equations in a system of linear equations is less than the number of unknowns in the system, then the system has infinitely many solutions.

is a false statement. Consider the example

$$\begin{aligned} x + y + z &= 0 \\ x + y + z &= 1 \end{aligned}$$

which consists of two equations in three unknowns ( $2 < 3$ ) but has no solutions, because it is inconsistent.

**Example 9:** To show a statement is *true*, it is not enough to illustrate it with one example. Consider the statement

If  $A$  is a  $2 \times 2$  matrix such that  $A^2 = I$ , then  $\det A = \pm 1$ .

which *is* a true statement. You can't prove it is true by simply considering one choice of  $A$ . Instead, you must give a general argument:

$$\begin{aligned} A^2 = I &\Rightarrow \det(A^2) = \det I \\ &\Rightarrow (\det A)^2 = 1 \\ &\Rightarrow \det A = \pm 1 \end{aligned}$$

**Inexcusable Errors:** Should any of the following types of fundamental algebraic errors show up in one of your solutions, everything after the appearance of the error will be forfeited. In computer jargon, these errors can all be considered “fatal errors.”

**Example 10:**  $\sqrt{a^2} = a$  is wrong! Correct is:  $\sqrt{a^2} = |a|$ .

**Example 11:**  $\sqrt{a^2 + b^2} = a + b$  is wrong. There just isn't any easy way to simplify this expression.

**Example 12:**  $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$  is wrong. Correct is

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}.$$

So,

$$\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b} \Leftrightarrow (a+b)^2 = ab \Leftrightarrow a^2 + ab + b^2 = 0,$$

which is *impossible* if  $a$  and  $b$  are non-zero real numbers.

**Example 13:**  $\ln\left(\frac{M}{N}\right) = \frac{\ln M}{\ln N}$  and  $\ln(MN) = \ln N \ln N$  are both wrong. Correct are

$$\ln\left(\frac{M}{N}\right) = \ln M - \ln N \text{ and } \ln(MN) = \ln M + \ln N,$$

if both  $M > 0$  and  $N > 0$ .

**Example 14:**  $\sin(\alpha + \beta) = \sin \alpha + \sin \beta$  and  $\cos(\alpha + \beta) = \cos \alpha + \cos \beta$  are both wrong. Correct are:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

and

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta.$$

**Example 15:**  $\sin(k\theta) = k \sin \theta$  is wrong, unless  $k = -1, 0, 1$ . There is no easy way to simplify this, but here are two special cases

$$\sin(2\theta) = 2 \sin \theta \cos \theta \text{ and } \sin(3\theta) = 3 \sin \theta - 4 \sin^3 \theta.$$

**Example 16:**  $\ln(M + N) = \ln M + \ln N$  is wrong. There just isn't any easy way to simplify  $\ln(M + N)$ .

**Absolute Value:** probably the most misunderstood formula from high school is the definition of absolute value:

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

That is, to simplify  $|a|$  you have to take cases.

**Example 17:** Thus  $|x|^2 = x^2$  and  $\sqrt{x^2} = |x|$  are correct, but  $|x|^3 = x^3$  is incorrect. Can you explain why?

**Some Comments About Marking:** If you make a copying error – or some other ‘dumb’ mistake – in your work which results in the rest of the problem being much easier than intended, then you will forfeit lots of marks. On the other hand, if you make a copying error and the rest of the problem remains of comparable difficulty, then you will only lose a few marks. Here is an example: consider the integral

$$\int \frac{1}{x^3 + 1} dx,$$

which is a fairly involved problem requiring partial fractions, completing the square and a trig substitution to solve. Say it is worth 10 marks on a test. If you miscopy the integral as

$$\int \frac{1}{x^3 - 1} dx$$

you could still get 9 out of 10, if the rest of your work is correct, because the exact same procedures as the intended question are involved. However, if you miscopy the question as

$$\int \frac{1}{x^2 + 1} dx$$

it becomes a one-liner, and you would get at most 1 mark out of 10.

**Some Comments About Your Calculator:** Don’t blame your calculator if it gives you the wrong answer. For example, many calculators are programmed to accept only positive arguments when taking roots of a number. So, even though there is a real value for

$$(-1)^{(1/3)},$$

some calculators will not evaluate this expression, or will display an answer of 0. The correct answer is of course

$$(-1)^{(1/3)} = -1.$$

Make sure you know how to use your calculator, and its built-in limits.

The same holds true for computer algebra systems, like *Matlab*, *Maple* or *Mathematica*. They aren’t always programmed as you might expect.