MAT188H1F LINEAR ALGEBRA: Syllabus for Fall 2018 as of November 15, 2018

2018-2019 Calendar Description:

> This course covers systems of linear equations and Gaussian elimination, applications; vectors in $\mathbb{R}^n$, independent sets and spanning sets; linear transformations, matrices, inverses; subspaces in $\mathbb{R}^n$, basis and dimension; determinants; eigenvalues and diagonalization; systems of differential equations; dot products and orthogonal sets in $\mathbb{R}^n$; projections and the Gram-Schmidt process; diagonalizing symmetric matrices; least squares approximation. Includes an introduction to numeric computation in a weekly laboratory.

This is a first course in linear algebra. Although this material is intrinsically simpler than calculus, it may *seem* more difficult because it is more abstract than calculus. In Calculus there are only *three* basic concepts: limits, derivatives and integrals. In linear algebra there are at least 50 new concepts, some more important than others, but all are new ideas that you must assimilate! However, once you get used to the new concepts you will find that the computations in this course are more routine than those in calculus. Indeed, much of what we will do in MAT188H1F is of an algorithmic nature. The other aspect of MAT188H1F that makes it more abstract than calculus is that we will expect you to be able to *prove* some things: proofs will be a part of lectures, homework *and* tests.

Section Instructors: by now you should be scheduled into one of the following Sections:

LEC0101 Elek	LEC0102 Guzman	LEC0103 Cluett	LEC0104 Uppal
LEC0105 Burbulla	LEC0106 Xiao	LEC0107 Kundu	LEC0108 Cohen

Textbooks: the references for the course will be two (very similar) on-line open-source free textbooks:

1. Nicholson's *Linear Algebra with Applications* (A1)
2. Kuttler's *Linear Algebra, A First Course* (A2)

Of the two textbooks, A2 is more standard in terms of the order of topics; A1 is less standard in that it covers eigenvalues and eigenvectors very early in the course. We shall regard A1 as our main reference, and follow it's order of topics. (We used Nicholson's book, or a slightly different version thereof, in the years 2007 to 2011.) A2 can be considered as an auxiliary reference which includes some examples and explanations that we may use to augment the material found in A1. Of course, you are welcome to read both; indeed you may find one author's style more to your liking than the other's. The notation used in the two books is for the most part identical; however, among the two books the notation for a vector includes all of the following:

$$X, \vec{x}, \mathbf{x}, \overrightarrow{OX}.$$

In terms of lectures it is easiest to use the "arrow notation," $\vec{x}$, for a vector, but you should be familiar with all the different notations.

In addition you should look over this **general advice** (GA) for first year math students.

Marking Scheme: Math Self-Assessment Quizzes: 2%; Practicals: 15%; Tutorials: 8%; Test 1: 15%; Test 2: 20%; Exam: 40%

Math Self-Assessment Quizzes: a link to the Math Self-Assessment Quizzes was emailed to all incoming Core 8 and TrackOne students on August 7, 2018. The seven quizzes, covering a review of high school math, are available until September 10. Your best 90 of 97 problems will count 2% towards your final mark in MAT188H1F.

Practicals: each student in MAT188H1F will be scheduled into a weekly lab covering numerical methods using Matlab. The labs are coordinated by Profs Stickel and Variawa. All inquiries related to the labs should be sent to mat188lab@gmail.com.

Test 1: a 100-minute term test is scheduled for Tuesday, October 2, between 1:15 and 2:55 PM in locations to be announced.

Test 2: a 100-minute term test is scheduled for Tuesday, November 6, between 1:15 and 2:55 PM in locations to be announced.

Final Exam: there will be a common final exam, 150 min long, during the exam period, Dec 7-21.

Classroom Deportment: the format of lectures is more formal than what you may be used to from high school. During lectures there should not be any disruptions that would prevent other students from hearing or seeing the instructor: no talking, no cell phones, no music, no eating or drinking. You should raise your hand to ask a question. You should arrive on time. If you do arrive late, please enter by a back door and sit down in the first available seat so as not to disrupt the rest of the class.

Calculators: use of a Casio FX-991 or Sharp EL-520 calculator will be permitted during all quizzes, tests and exams. However, it is still your responsibility to explain your work. A correct answer with no justification will receive no marks.

Course Coordinator: D. Burbulla. Office: GB 149

email: burbulla@math.toronto.edu; office hours: MTWR 12:10-2 PM; F 1:10-3 PM

Lecture Schedule: the Engineering term consists of 13 weeks; each week begins on a Thursday and ends on a Wednesday. The first day of classes is Thursday, September 6 and the last day of classes is Wednesday, December 5. Each lecturer will cover the indicated topics in his or her own style: not *everything* in the reference sections will necessarily be covered in lectures; instructors may become slightly ahead or behind schedule. In Week 5 seven lecture sections will miss a class due to the Thanksgiving holiday, so slightly less is scheduled for Week 5, compared to the other weeks. Note: all suggested homework questions are from Nicholson (A1), unless otherwise specified.

1. **Sep 6–12:** solving a system of linear equations using elementary operations; the augmented matrix of a system of linear equations; row operations; equivalent matrices; row echelon form of a matrix; reduced row echelon form of a matrix; Gaussian elimination; consistent systems and unique solutions; the rank of a matrix.

   Ref: A1 Sec 1.1, 1.2; A2 Sec 1.1, 1.2.

   Homework: Sec 1.1: #1(b), 4, 7(b), 10(b), 17, 19; Sec 1.2: #1(d),(f), 2(b), 3(b),(d), 5(d),(f),(h), 8(b), 9(b),(f), 12, 18

2. **Sep 13–19:** homogeneous systems of linear equations; trivial and non-trivial solutions; linear combinations of solutions; basic solutions; applications of systems of linear equations. Addition, scalar multiplication and transposition of matrices; symmetric matrices.

   Ref: A1 Sec 1.3, 1.4, 1.5, 1.6, 2.1; A2 Sec 1.2

   Homework: Sec 1.3: #1, 2(d), 3(b),(d), 5(b),(d), 7(b),(d), 11; Sec 1.4: #1, 1; Sec 1.6: #2, 4;

   Sec 2.1: #1(b), 2(h), 6(b), 8(b), 13, 14(d), 15(d), 19, 20, 21

3. **Sep 20–26:** the algebra of vectors and matrices: addition and scalar multiplication of vectors in $\mathbb{R}^n$; systems of equations and matrix-vector multiplication; introduction to matrix transformations; matrix multiplication; composition of matrix transformations.

   Ref: A1 Sec 2.2, 2.3 omit Block Multiplication and Directed Graphs; A2 Sec 2.1

   Homework: Sec 2.2: #1(b), 2(b), 3(d), 4, 5(b),(d), 6, 8(b), 10, 11, 12, 13, 18;

   Sec 2.3: #1(b),(d),(f),(h),(j), 3(b), 4(b), 6(b), 7(b), 8(b), 16(d), 20, 22(b), 27, 32, 34

4. **Sep 27–Oct 3:** matrix inverses; formula for the inverse of a 2×2 matrix; properties of inverses; the Inverse Matrix Theorem; linear transformations and matrix transformations; the inverse of a matrix transformation.

   Ref: A1 Sec 2.4, 2.6; A2 Sec 5.2, 5.3, 5.4

   Homework: Sec 2.4: #2(d),(h),(i), 3(b),(d), 4(b), 5(b),(d),(h), 9, 12, 15(b), 16, 24(b), 25(b), 33(b), 34, 39(b); Sec 2.6: #1(b), 3, 4, 7(b), 8, 12, 14, 18

5. **Oct 4–10:** cofactors; determinants; properties of determinants; the Product Theorem. Optional topics: adjugate of a matrix, a formula for the inverse of a matrix, Cramer's Rule.

   Ref: A1 Sec 3.1, 3.2, for the most part without proofs; A2 Sec 3.1, 3.2

   Homework: Sec 3.1: #1(d),(f),(j),(n), 5(b), 6(b), 8(b), 9, 15(b), 16(b),(d), 26;

   Sec 3.2: #2(b),(d),(f), 3(b), 4(b), 6(b), 10, 16, 20,

6. **Oct 11-17:** introduction to eigenvalues and eigenvectors; the characteristic polynomial of a matrix; matrix diagonalization; powers of a matrix; linear dynamical systems, if time permits.

   Ref: A1 Sec 3.3; A2 Sec 7.1, 7.2

   Homework: Sec 3.3: #1(b),(d),(f),(h), 2(b), 8(b), 9, 12, 14, 20(b), 23(b), 27

7. **Oct 18–24:** other applications of diagonalization: systems of linear differential equations; linear recurrence relations. Vectors in $\mathbb{R}^n$: length of vectors; vector addition and scalar multiplication; geometry of vectors; lines and vectors.

   Ref: A1 Sec 3.4, 3.5, 4.1; A2 Sec 4.1, 4.2, 4.3, 4.4, 4.5, 4.6

   Homework: Sec 3.4: #1(b),(d), 9; Sec 3.5: 1(b),(d), 6; Sec 4.1: #1(b),(d),(f), 2(b), 6(b), 7(b),(d), 9(b),(d),(f), 10(b), 11(b), 17(b), 20(b), 21, 22(b),(d),(f), 23(b), 24(b),(d), 29

8. **Oct 25–31:** dot products; properties of the dot product; angle between vectors; orthogonal vectors; projections; vector and normal equations of a plane; cross products; properties of the cross product.

   Ref: A1 Sec 4.2, 4.3; A2 Sec 4.7, 4.8, 4.9

   Homework: Sec 4.2: #2(b),(d),(f), 4(b), 6, 8(b), 10(b),(d), 11(b),(d), 12(b), 13(b),(d), 14(b),(d),(f),(h), 15(b),(d),(f), 16(b), 19(b), 23(b), 24(b),(d), 34, 37, 38; Sec 4.3: #2, 3(b), 4, 5, 6(b), 8, 10

9. **Nov 1–7:** linear operators on $\mathbb{R}^2$ and $\mathbb{R}^3$: reflections, projections and rotations. Subspaces of $\mathbb{R}^n$; spanning sets.

   Ref: A1 Sec 4.4, 5.1; A2 Sec 4.10

   Homework: Sec 4.4: #1, 2(b), 3(b),(d),(f), 6;

   Sec 5.1: 1(b),(d),(f), 2(b),(d), 3(b), 8, 10, 12, 13, 14, 15(b), 16, 17(a), 22

10. **Nov 8-14:** linear independence in $\mathbb{R}^n$; basis and dimension of a subspace; orthogonality.

    Ref: A1 5.2. 5.3; A2 Sec 4.10

    Homework: Sec 5.2: #1(b),(d), 2(b),(d), 3(b),(d), 4(b),(d),(f), 5(b), 6(b),(d),(f), 7, 8, 10, 12, 14, 15, 17(b); Sec 5.3: #1(b), 2, 3(b),(d), 4(b), 5(b), 6(d), 7, 12, 13, 14, 15

11. **Nov 15-21:** the rank of a matrix revisited; the subspaces determined by a matrix; similarity and diagonalization.

    Ref: A1 Sec 5.4, 5.5; A2 Sec 4.10

    Homework: Sec 5.4: #1(b),(d), 2(b),(d), 3, 4, 5, 7(b), 9, 10, 15, 18;

    Sec 5.5: #1(b),(d),(f), 3(b), 4(b),(d), 7, 10, 13(b), 16

12. **Nov 22-28:** least squares approximations; orthogonal complements and projections; Gram-Schmidt algorithm;

    Ref: A1 Sec 5.6, 8.1 without Theorem 4; A2 Sec 4.11,

    Homework: Sec 5.6: #1(b), 2(b),(d), 3(b); Sec 8.1: #2(b),(d),(f), 3, 4(b),(d), 5, 10, 11, 16

13. **Nov 29–Dec 5:** symmetric matrices and orthogonal diagonalization; review.

    Ref: A1 Sec 8.2; A2 Sec 7.4

    Homework: Sec 8.2: #1(f),(h), 5(b),(d),(f), 6, 15, 18, 19, 23, 24(a),(b)

Tutorial Schedule: each tutorial will meet 12 times during the term. Tutorials begin on Monday, September 10 and end on Monday, December 3. The University is closed on Thanksgiving Monday, October 8. So the first four tutorial cycles run from Monday, September 10 to Friday, October 5; the last eight tutorial cycles run from Tuesday, October 9 until Monday, December 3. To get marks for tutorial quizzes and/or assignments, you must attend your own tutorial. Only the best 4 of 6 tutorial quizzes will be counted.

1. **Sep 10–14:** Proof

   Ref: GA and A1 Appendix B

2. **Sep 17–21:** Quiz 1

   Ref: A1 Sections 1.1, 1.2

3. **Sep 24–28:** Quiz 2

   Ref: A1 Sections 1.3, 1.4, 1.6 and 2.1

4. **Oct 1–5:**

   Ref: A1 Sections 2.2, 2.3

5. **Oct 9–15:**

   Ref: A1 Sections 2.4, 2.6

6. **Oct 16–22:** Quiz 3

   Ref: A1 Sections 3.1, 3.2

7. **Oct 23–29:** Quiz 4

   Ref: A1 Sec 3.3

8. **Oct 30–Nov 5:**

   Ref: A1 Sec 3.5

9. **Nov 6–12:**

   Ref: A1 Sections 4.1, 4.2, 4.3 and (4.4?)

10. **Nov 13–19:**

    Ref: A1 Sections 5.1, 5.2

11. **Nov 20–26:** Quiz 5

    Ref: A1 Sections 5.1, 5.2

12. **Nov 27–Dec 3:** Quiz 6

    Ref: A1 Sections 5.3, 5.4

Learning Outcomes (short version):

1. Write well-written, well-explained solutions to given problems using correct notation
2. Apply the algorithms of linear algebra to specific examples and to practical problems
3. Solve geometric problems in 3 dimensions using properties of vectors and determinants
4. Prove statements about matrices & vectors by using properties of matrices and vectors
5. Apply methods of linear algebra to analyze a real-world situation without cues

Learning Outcomes (long version):

- Given a problem, be it as simple as a calculation or as complicated as a long, involved word problem, students should be able to write a well-organized solution that defines any variables used, describes any assumptions made, includes diagrams that illustrate the connection between variables, uses correct mathematical notation, and provides full explanation of all the steps involved. In particular, students should be able to give complete solutions to the following types of problems:
    1. any word problem that can be reduced to a system of linear equations
    2. any word problem that can be reduced to a system of linear differential equations
- In the real vector space $\mathbb{R}^n$, students should be able to
    1. define a linear combination of m vectors in $\mathbb{R}^n$
    2. define and calculate the dot product of two vectors
    3. define and calculate the length of a vector
    4. define and calculate the projection of one vector onto another non-zero vector
    5. define and calculate the cross product of two vectors in $\mathbb{R}^3$
    6. calculate the area of the parallelogram determined by two vectors in $\mathbb{R}^3$
    7. calculate the volume of the parallelepiped determined by three vectors in $\mathbb{R}^3$
    8. solve geometric problems in $\mathbb{R}^3$ that involve finding the minimum distance
        (*i*) from a line to a point not on the line
        (*ii*) from a plane to a point not on the plane
        (*iii*) between two skew lines, or between two parallel lines
    9. prove statements about vectors, using properties of vector addition, scalar multiplication, the length of a vector, the dot product, projections, and the cross product
- Given a system of m linear equations in n variables, students should be able to
    1. write down the augmented matrix of the system
    2. solve the system by using Gaussian elimination
- Given a subset S of $\mathbb{R}^n$ students should be able to determine
    1. whether S is linearly independent or not
    2. whether S is orthogonal or not

3. whether S is orthonormal or not
4. whether S is a subspace of $\mathbb{R}^n$ or not

- Given a subspace S of $\mathbb{R}^n$ students should be able to find
  1. a spanning set for S
  2. a basis for S
  3. an orthogonal basis for S, using the Gram-Schmidt algorithm, if necessary
  4. the dimension of S
  5. a basis for $S^{\perp}$, the orthogonal complement of S
  6. the dimension of $S^{\perp}$
  7. the projection of a vector $\vec{x}$ onto S, and the projection of $\vec{x}$ onto $S^{\perp}$
- Given matrices of appropriate sizes, students should be able to
  1. add them, subtract them, multiply them, calculate their transposes, and multiply them by a scalar
  2. solve problems and prove statements about matrices, using properties of matrix addition, matrix multiplication, and transposition
- Given an $m \times n$ matrix A students should be able to
  1. calculate the reduced row echelon form of A
  2. find the rank of A
  3. find a basis for, and dimension of, the row space of A, the column space of A, and the null space of A
  4. state the Rank-Nullity Theorem
  5. solve problems and prove statements about matrices, using the Rank-Nullity Theorem
- Given an $n \times n$ matrix A students should be able to
  1. calculate $\det(A)$ by using suitable row and column operations, plus cofactor expansions
  2. determine if A is invertible, and if it is calculate A^{-1}, the inverse of A
  3. solve problems and prove statements about determinants, using properties of determinants
  4. determine whether A is orthogonal or not
  5. state and make use of various properties of A that are equivalent to the statement "A is invertible"
  6. solve the matrix equation $A\vec{x} = \vec{b}$ for $\vec{x}$ by using A^{-1} if A is invertible
  7. define an eigenvalue and an eigenvector of A
  8. calculate the characteristic polynomial of A
  9. find the eigenvalues of A and a basis for each eigenspace of A
  10. diagonalize A, or explain why it is not diagonalizable
  11. define what it means for the $n \times n$ matrix B to be similar to A
  12. calculate powers of A if A is diagonalizable

13. solve the linear system of differential equations, $\frac{d\vec{y}}{dt} = A\vec{y}$, for $\vec{y}$ if A is diagonalizable
14. determine whether A is symmetric or not, and if it is, orthogonally diagonalize A
15. solve problems and prove statements about matrices, using properties of eigenvalues and eigenvectors

- Given a set of data points in $\mathbb{R}^2$, students should be able to find the least squares approximation linear, quadratic (or other) model that best fits the data, by making use of the normal equations

- Given a function $T : \mathbb{R}^m \longrightarrow \mathbb{R}^n$, students should be able to determine whether or not T is a linear transformation

- Given a linear transformation $L : \mathbb{R}^m \longrightarrow \mathbb{R}^n$ students should be able to

  1. calculate the standard matrix of L
  2. calculate $L + M$, $a\,L$ and $L \circ N$, where a is a scalar in $\mathbb{R}$, and M and N are other linear transformations
  3. calculate the standard matrices of $L + M$, $a\,L$ and $L \circ N$ in terms of the matrices of L, M and N

- Students should be able to solve problems and prove statements about linear transformations, using properties of linear transformations

- Given a linear transformation $L : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ students should be able to

  1. draw the image of the unit square under L and calculate the area of the image of the unit square in terms of the determinant of the matrix of L
  2. interpret geometrically the action of L, if L is a dilation, a stretch, a shear, a projection, a rotation or a reflection in a line
  3. calculate the area of the image of a region $\mathcal{R}$ under L in terms of the area of $\mathcal{R}$ and the determinant of the matrix of L

- Given a linear transformation $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ that represents a projection onto a subspace of $\mathbb{R}^3$, or a reflection in a plane in $\mathbb{R}^3$, students should be able to calculate the matrix of L

- Matlab stuff . . .