MAT186H1F CALCULUS I: Syllabus for Fall 2018 as of November 22, 2018

2018-2019 Calendar Description:

> Topics include: limits and continuity; differentiation; applications of the derivative—related rates problems, curve sketching, optimization problems, L'Hopital's rule; definite and indefinite integrals; the Fundamental Theorem of Calculus; applications of integration in geometry, mechanics and other engineering problems.

A lot of the material will be review of high school material, but not *all* of it; you must pay attention and assimilate the new material as well as the old. We do assume that everything about integration is new to you. To intersperse the new and the old, we will introduce some of the material about integration as soon as possible.

Section Instructors: by now you should be scheduled into one of the following Sections:

LEC0101 Ko	LEC0102 Fusca	LEC0103 Cohen	LEC0104 Burbulla
LEC0105 Jie	LEC0106 Bischoff	LEC0107 Pham	LEC0108 Matviichuk

Textbooks: the references for the course will be three on-line open-source free textbooks:

1. APEX Calculus I, U of T version (C1)
2. Active Calculus 2.0 (C2)
3. Calculus: Early Transcendentals (C3)

C1 is for the first term only; it is part of the book that will be used in MAT187H1S. The whole book, which will be used for Calculus I and II, is APEX Calculus, U of T version. C1 is our principal reference, but it has no review of high school material, whereas C3 does. C2, in its html format, is notable for its WeBWorK interactive exercises, so is a good source of some practice problems. In the first week of lectures, the main reference will be C3; in weeks 3, 5, 6, 7, 8, 9, 10, 11 and 12 the main reference will be C1; in week 13, the main reference will be C2. In weeks 2 and 4, in which we "jump around" between differential and integral calculus, the best reference will probably be your lecture notes.

In addition you should look over this **general advice** (GA) for first year math students.

Marking Scheme: Diagnostic Tests: 5%; Tutorials: 10%; Test 1: 20%; Test 2: 20%; Exam: 45%

Diagnostic Tests: to be written on Tuesday, September 11, between 1:10 and 3 PM. The purpose of these tests is to determine if any students have gaps in their high school math. Then we can offer help to these students before the *real* tests start. The Math Diagnostic Test is a 110 min multiple choice test covering six sections: Algebra, Functions, Graphing, Trigonometry, Exponentials and Logarithms, and Calculus Fundamentals. No calculators are permitted. The locations for the test are alphabetically by the first letter of you surname:

Surname	Location
A to I	EX200
J to R	EX100
S	EX300
T to W	EX310
X to Z	EX320

EX is the Examination Centre, 255 McCaul Street.

This test will count 5% of your final mark in MAT186H1F. Your mark is based on the five parts, Algebra, Functions, Graphing, Trigonometry, Exponentials & Logarithms, with each part to count 1%. (The sixth part, Calculus Fundamentals will not count, its just for our interest.) To get your 1% for each part you must get at least 8 of the 10 questions correct; if you get 7 or less, then your mark for that part is 0%. If you get 7 or less on any of the five parts, then you can rewrite those parts two weeks later, on September 25. There will also be some help sessions made available to students who want more practice on any of the five parts of the Diagnostic Tests before September 25.

Test 1: a 100-minute term test is scheduled for Tuesday, October 16, between 1:15 and 2:55 PM in locations to be announced.

Test 2: a 100-minute term test is scheduled for Tuesday, November 20, between 1:15 and 2:55 PM in locations to be announced.

Final Exam: there will be a common final exam, 150 min long, during the exam period, Dec 7-21.

Calculators: use of a Casio FX-991 or Sharp EL-520 calculator will be permitted during all quizzes, tests and exams, except for the Math Diagnostic Tests. However, it is still your responsibility to explain your work. A correct answer with no justification will receive no marks.

Course Coordinator: D. Burbulla. Office: GB 149

email: burbulla@math.toronto.edu; office hours: MTWR 12:10-2 PM; F 1:10-3 PM

Lecture Schedule: the Engineering term consists of 13 weeks; each week begins on a Thursday and ends on a Wednesday. The first day of classes is Thursday, September 6 and the last day of classes is Wednesday, December 5. Each lecturer will cover the indicated topics in his or her own style: not *everything* in the reference sections will necessarily be covered in lectures. Some weeks are almost entirely review of high school material; some weeks cover only new material; and some weeks combine review with the new. Instructors may become slightly ahead or behind schedule. In Week 5 three lecture sections will miss a class due to the Thanksgiving holiday; this may result in some lecture sections becoming one hour behind the following schedule.

1. **Sep 6–12:** inverse functions; review of logarithmic, exponential and trigonometric functions; sum and difference formulas for sine and cosine; double angle formulas; definition of the inverse trigonometric functions $\sin^{-1}, \cos^{-1}, \tan^{-1}, \sec^{-1}$; the hyperbolic functions.

   Ref: C3 Ch 1: *Review,* and C3 Ch 2: *Functions.* Also, C1 Figure 2.7.3, on p 125, which defines the six inverse trigonometric functions, as we shall use them. NB: C3 Exercises 4.8.3 and 4.8.4 describe *different* definitions of $\sec^{-1}$ and $\csc^{-1}$; we shall *never* use them!

   Homework: C3 all exercises in Sec 1.3, 2.4, 2.5, 2.6, 2.7 (without #2.7.6 and #2.7.7)

2. **Sep 13–19:** the tangent line problem and the area problem: definition of the tangent line to a curve as a limit of secant lines to the curve; average velocity and instantaneous velocity; the idea of a limit of a function; the idea of a derivative; definition of the area under a curve as the limit of a sum of rectangular approximations to the area under the curve; Riemann sums; the definite integral; velocity and displacement.

   Ref: C1 Sec 2.1, 2.2, 5.3; C2 Sec 1.1, 1.2, 1.3, 4.1, 4.2; C3 Sec 4.1, 6.1.

   Homework: C1 Sec 1.1: #19, 20, Sec 5.3: #29-34. C3 Sec 4.1: #1-11, Sec 6.1: #6.1.7. 6.1.8.

3. **Sep 20–26:** informal definition of a limit; limit laws; basic algebraic techniques for calculating a limit in the 0/0 form—factoring and rationalizing; the squeeze law; one-sided limits; infinite limits and vertical asymptotes of a function; limits at infinity and horizontal asymptotes of a function.

   Ref: C1 Sec 1.1, 1.3, 1.4, 1.6; C2 Sec 1.2, 1.3; C3 Sec 3.1, 3.3, 3.4, 3.5, 3.6

   Homework: C3 Exercises #3.3.1, 3.4.1, 3.5.1, 3.5.2, 3.5.4, 3.5.5, 3.6.1, 3.6.2, 3.6.3, 3.6.4, 3.6.5

4. **Sep 27–Oct 3:** continuity of a function at a point; types of discontinuity—infinite, jump and removable; continuity on a closed interval; the Intermediate Value Theorem; isolating solutions to an equation by using IVT; definition of the derivative; differentiability of f at a implies continuity of f at a; definition of the Riemann Integral; basic properties of the Riemann Integral; continuity of f on the interval $[a, b]$ implies existence of the definite integral of f on $[a, b]$.

   Ref: C1 Sec 1.5, 5.2; C2 Sec 1.3, 1.7, 4.1, 4.2, 4.3; C3 Sec 3.7, 4.2, 6.1

   Homework: C1 Sec 1.5 Exercises #11-38, Sec 5.2 Exercises #19-26. C3 Exercises # 3.7.3, 3.7.6

5. **Oct 4–10:** the derivative function; slopes of tangent lines and normal lines to a curve; e and the derivative of exponential functions; rules of differentiation—derivatives of a sum, difference, product and quotient; higher order derivatives; derivatives of the trigonometric functions; derivatives as rates of change; position, velocity, acceleration; the chain rule.

   Ref: C1 Sec 2.1, 2.2, 2.3, 2.4, 2.5; C2 Sec 2.1, 2.2, 2.3, 2.4, 2.5; C3 Sec 4.2, 4.3, 4.4, 4.5.

   Homework: C1 Sec 2.3 Exercises #16, 28, 31, 33, 37, Sec 2.4 Exercises #16, 19, 20, 29, 39, 44, Sec 2.5 Exercises #9, 18, 30, 32, 35, 39

6. **Oct 11-17:** implicit differentiation; related rates problems; derivatives of logarithmic and exponential functions; logarithmic differentiation; derivatives of the inverse trigonometric functions; derivatives of inverse functions in general.

   Ref: C1 Sec 2.6, 2.7, 4.2; C2 Sec 2.6, 2.7, 3.5; C3 Sec 4.6, 4.7, 4.8, 5.1.

   Homework: C1 Sec 2.6 Exercises #20, 26, 29, 32, 38, 39; Sec 2.7 Exercises #11, 12, 22, 23, 28; C3 Sec 5.1 Exercises #5.1.3, 5.1.4, 5.1.5, 5.1.6, 5.1.8, 5.1.9, 5.1.10, 5.1.11, 5.1.12, 5.1.13

7. **Oct 18–24:** antiderivatives; the indefinite integral; basic indefinite integration formulas; position, velocity, acceleration revisited; introduction to differential equations and initial value problems; exponential growth and decay; the Fundamental Theorem of Calculus—evaluating and differentiating definite integrals.

   Ref: C1 Sec 5.1, 5.2, 5.3, 5.4; C2 Sec 4.3, 4.4, 5.2; C3 Sec 6.1, 6.2, 6.3.

   Homework: C1 Sec 5.1 Ex. #29-39, Sec 5.2 #7, 9, 11, 13, Sec 5.4 Ex. #5-28, 41-50, 55-58

8. **Oct 25–31:** maximum and minimum values; the Extreme Value Theorem; critical points; Rolle's Theorem; the Mean Value Theorem and its consequences; increasing and decreasing functions; the first derivative test; concavity, inflection points, and the second derivate test; curve sketching; optimization problems.

   Ref: C1 Sec 3.1, 3.2, 3.3, 3.4, 3.5, 4.3; C2 Sec 3.1, 3.2, 3.3, 3.4; C3 Sec 5.2, 5.3, 5.6, 5.7.

   Homework: C1 Sec 3.1 #7, 8,17-26. Sec 3.5 #15-26. C3 Sec 5.6 #58 ; Sec 5.7 #1-20

9. **Nov 1–7:** linear approximations; Newton's Method for approximating solutions to the equation $f(x) = 0$; L'Hopital's Rule and indeterminate forms.

   Ref: C1 Sec 4.1, 4.4; C2 Sec 2.8; C3 Sec 5.4, 5.5.

   Homework: C1 Sec 4.1 #9, 10, 12, 14; C2 Sec 1.8 #1, 2, 3; C3 Sec 5.5 #5.5.15-5.5.21

10. **Nov 8-14:** integrating even and odd functions; average value of a function; Mean Value Theorem for Integrals; change of variables in indefinite integrals; change of variables in definite integrals; integration by parts.

    Ref: C1 Sec 5.4, 5.5, 5.6; C2 5.3, 5.4; C3 7.1, 7.4, 8.4.

    Homework: C3 #7.1.1-7.1.19, #7.4.6-7.4.13, #8.4.1-8.4.5; C1 Sec 5.5 #79-86, Sec 5.6 #24, 25

11. **Nov 15-21:** velocity and net change of position; average velocity and average speed; areas of regions between curves—integrating with respect to x or y; volume by slicing; solids of revolution; volumes by disc and washer methods.

    Ref: C1 Sec 5.2, 6.1, 6.2; C2 Sec 6.1, 6.2; C3 Sec 8.1, 8.2, 8.3.

    Homework: C3 #8.4.5; C1 Sec 6.1 #21-26, Sec 6.2 #13 -21

12. **Nov 22-28:** volumes by shells; arc length; surface area of a solid of revolution; density and mass; work.

    Ref: C1 Sec 6.3, 6.4, 6.5; C2 Sec 6.1, 6.2, 6.3, 6.4; C3 Sec 8.3, 8.5, 8.6, 8.7, 8.8.

    Homework: C1 Sec 6.3 #13-18, Sec 6.4 #5, 7, 9, 11, 29-33, Sec 6.5 #16, 24, 26

13. **Nov 29–Dec 5:** functions defined in terms of definite integrals; the natural logarithm function defined as a definite integral; review

    Ref: C2 Sec 5.1 **Homework:** C2 Section 5.1 #3-7

Tutorial Schedule: each tutorial will meet 12 times during the term. Tutorials begin on Monday, September 10 and end on Monday, December 3. The University is closed on Thanksgiving Monday, October 8. So the first four tutorial cycles run from Monday, September 10 to Friday, October 5; the last eight tutorial cycles run from Tuesday, October 9 until Monday, December 3. To get marks for tutorial quizzes and/or assignments, you must attend your own tutorial. Only the best 5 of 7 tutorial quizzes will count.

1. **Sep 10–14:**

   Ref: C3 Sections 2.3, 2.4 and 2.5

2. **Sep 17–21:** Quiz 1

   Ref: C3 Sections 1.3 and 2.6

3. **Sep 24–28:** Quiz 2

   Ref: C2 Sections 1.1, 1.3, 4.1 and 4.2

4. **Oct 1–5:** Quiz 3

   Ref: C3 Sections 3.1, 3.3, 3.4, 3.5 and 3.6; C1 Sections 1.1, 1.3, 1.4 and 1.6.

5. **Oct 9–15:** Quiz 4

   Ref: C1 Section 1.5

6. **Oct 16–22:**

   Ref: C1 Sections 2.1, 2.2, 2.3, 2.4, 2.5

7. **Oct 23–29:**

   Ref: C1 Sections 2.6, 2.7, 4.2; C3 Sections 4.7, 4.8, 5.1

8. **Oct 30–Nov 5:**

   Ref: C1 Sections 5.1, 5.2, 5.4

9. **Nov 6–12:** Quiz 5

   Ref: C1 Sec 3.1, 3.2, 3.3, 3.4, 3.5, 4.3

10. **Nov 13–19:** Quiz 6

    Ref: C1 Sec 4.1, 4.4; C3 Sec 5.4, 5.5

11. **Nov 20–26:**

    Ref: C1 Sec 5.4, 5.5, 5.6

12. **Nov 27–Dec 3:** Quiz 7

    Ref: C1 Sec 5.2, 6.1, 6.2

Learning Outcomes (short version): students should be able to do the following:

1. Define limit, derivative and definite integral; and be able to calculate each of them.
2. Write well-written, well-explained solutions to given problems using correct notation.
3. Given the graph of one of f, f' or f", figure out the graphs of the other two.
4. Apply methods of calculus to solve optimization, related rates and physics problems.
5. Apply methods of calculus to calculate lengths, areas, volumes and average values.
6. Apply concepts and methods of calculus to analyze a real-world situation without cues.

Learning Outcomes (long version):

- Given a problem, be it as simple as a calculation or as complicated as a long, involved word problem, students should be able to write a well-organized solution that defines any variables used, describes any assumptions made, includes diagrams that illustrate the connection between variables, uses correct mathematical notation, and provides full explanation of all the steps involved. In particular, students should be able to give complete solutions to the following types of word problems:
  1. related rates problems
  2. optimization problems that can be reduced to one independent variable
  3. physics problems that involve position, velocity, acceleration, density, mass or work
  4. initial value problems with one differential equation in terms of one independent parameter
  5. exponential growth and decay problems
- Given a function $y = f(x)$, students should be able to
  1. determine if f is one-to-one, and if it is one-to-one, to calculate the inverse of f
  2. determine if f is even or odd, or neither
  3. find and classify all the discontinuities of f
  4. define the derivative of f at $x = a$
  5. define and calculate the equations of the tangent line and the normal line to f at $x = a$
  6. use the tangent line of f at $x = a$ to obtain a linear approximation of f near a
  7. solve geometric problems that involve tangent lines and normal lines of f
  8. calculate the first and second derivative of f
  9. find all the asymptotes—vertical, slant or horizontal—to the graph of f
  10. find all the critical points and inflection points of f and determine if the critical points are maximum points or minimum points of f
  11. find the intervals on which f is increasing, decreasing, concave up or concave down
  12. sketch the graph of f, labelling all the asymptotes, maximum or minimum points, inflection points, and vertical tangents of f
  13. approximate roots of the equation $f(x) = 0$, using Newton's method
  14. define an antiderivative of f on the interval $[a, b]$

15. define the indefinite integral of f
16. find the indefinite integral of f by using appropriate algebraic manipulations or by making a change of variable, or by using integration by parts, or a combination of all of these
17. define the definite integral of f on the interval $[a, b]$
18. approximate the definite integral of f on the interval $[a, b]$ by using Riemann sums
19. calculate the definite integral of f on the interval $[a, b]$ by using the Fundamental Theorem of Calculus
20. solve problems and prove statements about definite integrals using properties of definite integrals
21. calculate the average value of f on the interval $[a, b]$
22. calculate the length of the curve $y = f(x)$ on the interval $[a, b]$
23. calculate the surface area of the solid of revolution obtained by revolving $y = f(x)$, on the interval $[a, b]$, about the x-axis, or about the y-axis
24. use the method of discs to calculate the volume of the solid of revolution obtained by revolving $y = f(x)$, on the interval $[a, b]$, about an axis parallel to the x-axis
25. use the method of shells to calculate the volume of the solid of revolution obtained by revolving $y = f(x)$, on the interval $[a, b]$, about an axis parallel to the y-axis

- Given a limit in one of the following indeterminate forms, $0/0, \infty/\infty, \infty - \infty, 0 \cdot \infty, 1^\infty, 0^0$ or ∞^0, students should be able to calculate the limit by factoring, rationalizing, or other algebraic manipulation; or by applying L'Hopital's Rule or the Squeeze Law, as appropriate

- Given the graph of any one of f, f' or f'', students should be able to

  1. figure out possible graphs of the other two
  2. decide whether or not a given graph is a possible graph for either of the other two

- Given the velocity function in terms of time t of a particle moving along a straight line, students should be able to set up and evaluate definite integrals to find

  1. the net change of position of the particle on the time interval $[a, b]$
  2. the total distance travelled by the particle on the time interval $[a, b]$
  3. the average velocity of the particle on the time interval $[a, b]$
  4. the average speed of the particle on the time interval $[a, b]$

- Given a region in the xy-plane, bounded by one or more curves, students should be able to

  1. find the area of the region by setting up and evaluating one or more appropriate definite integrals with respect to x, or with respect to y, whichever is more suitable
  2. find the volume of the solid of revolution obtained by revolving the region about a given axis parallel to the x- or y-axis, by setting up and evaluating one or more appropriate definite integrals with respect to x, or with respect to y, whichever is more suitable