

OLYMON

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Please send your solution to

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no later than November 30, 2006. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

PROBLEMS FOR OCTOBER

458. Let ABC be a triangle. Let A_1 be the reflected image of A with axis BC , B_1 the reflected image of B with axis CA and C_1 the reflected image of C with axis AB . Determine the possible sets of angles of triangle ABC for which $A_1B_1C_1$ is equilateral.
459. At an International Conference, there were exactly 2006 participants. The organizers observed that: (1) among any three participants, there were two who spoke the same language; and (2) every participant spoke at most 5 languages. Prove that there is a group of at least 202 participants who speak the same language.
460. Given two natural numbers x and y for which

$$3x^2 + x = 4y^2 + y ,$$

prove that their positive difference is a perfect square. Determine a nontrivial solution of this equation.

461. Suppose that x and y are integers for which $x^2 + y^2 \neq 0$. Determine the minimum value of the function

$$f(x, y) \equiv |5x^2 + 11xy - 5y^2| .$$

462. For any positive real numbers a, b, c, d , establish the inequality

$$\sqrt{\frac{a}{b+c}} + \sqrt{\frac{b}{c+d}} + \sqrt{\frac{c}{d+a}} + \sqrt{\frac{d}{a+b}} > 2 .$$

463. In *Squareland*, a newly-created country in the shape of a square with side length of 1000 km, there are 51 cities. The country can afford to build at most 11000 km of roads. Is it always possible, within this limit, to design a road map that provides a connection between any two cities in the country?
464. A square is partitioned into non-overlapping rectangles. Consider the circumcircles of all the rectangles. Prove that, if the sum of the areas of all these circles is equal to the area of the circumcircle of the square, then all the rectangles must be squares, too.