Homer Simpson and Fermat's Last Theorem

Students who do well in mathematics and physics do not always wind up in academia or some research institute. David S. Cohen who graduated in physics from Harvard joined the writing team for the television series, *The Simpsons.* However, this did not mean that he left his scientific interests behind, for he insinuated various mathematical and physical formulae into the background in some of the episodes.

For example, in the episode *Treehouse of Horrors VI*, which aired on October 29, 1995, we see written on a blackboard the equation:

$$1782^{12} + 1841^{12} = 1922^{12}$$

If this equation were true, its discovery would be momentous. It is related to a longstanding conjecture due to a seventeenth century French mathematician, Pierre Fermat (1601-1665). Fermat noted that there are lots of squares that are equal to the sum of two other squares, such as $5^2 = 3^2 + 4^2$ and $17^2 = 8^2 + 15^2$. And then he claimed without giving a proof that the same was not true for higher powers of whole numbers: no cube was the sum of two nonzero cubes, no fourth power was equal to the sum of two nonzero fourth powers, and so on. This assertion, written in the margin of one of his mathematics books and discovered after his death, is accompanied by one of the most notorious statements in the history of mathematics: *Cuius rei demonstrationem mirabilem sane detexi, hanc marginis exiguitas non ca-peret*. This simply states that he has a proof but the margin is too small to contain it. No proof was found among his papers. For the next three hundred years, mathematicians struggled to either prove this or find an example that showed the statement to be false. By the middle of the nineteenth century, Fermat's result had been demonstrated for many particular cases, but not for every possible power. Homer Simpson seems to have found a counterexample for twelfth powers. However, an accurate calculator quickly shows that the equation is false.

The left side is equal to

$$2,541,210,258,614,589,\cdots$$

and the right side to

$$2,541,210,259,314,801,\cdots$$

Both numbers have 40 digits, and, as you can see, agree to the first nine digits. However, we don't need to go to all that trouble to disprove the equation. The number 1782^{12} is an even number, while the number 1841^{12} is odd. Their sum must be odd, so cannot possibly be 1922^{12}.

In the episode *The Wizard of Evergreen Terrace*, telecast on September 20, 1998, Homer has another go, and comes up with

$$3987^{12} + 4365^{12} = 4472^{12}.$$

Alas, this again turns out to be false. Can you see why? Unfortunately, the parity test (checking for even and odd) does not work. We might think of checking the last digit of the three numbers. The first number on the left side ends in 1, the second in 5. However, the number of the right side does end in 6, so this not help. However, the two numbers on the left side are both odd squares and so leave a remainder 1 upon division of 4. The remainder when you divide the left side by 4 is therefore 2. However, the right side is a multiple of 4.

You can find out some more details about Homer Simpson's mathematical adventures by firing up *http://boingboing.net/2014/10/17/homers-last-theorem.html*.

A proof of Fermat's Theorem was published in 1995. It was due to the Princeton mathematician, Andrew Wiles, who cracked it after ten years painstaking work using modern and sophisticated methods. However, an historical puzzle still remains. Did Fermat have a proof? And if so, what was it? Considering the the problem was open for over 300 years, it is highly unlikely that Fermat could have settled it given the state of mathematical knowledge at the time. But he might have had some invalid argument, which is now lost.

Problems such as this lead to all sorts of other interesting questions. While no nonzero cube is the sum of two nonzero cubes, there are infinitely many cubes that can be expressed as the sum of three cubes, such as: $6^3 = 3^3 + 4^3 + 5^3$; $9^3 = 1^3 + 6^3 + 8^3$; $12^3 = (-1)^3 + 9^3 + 10^3$. The eighteenth century mathematician, Leonard Euler (1707-1783) conjectured that there were nth powers that could be expressed as the sum of n such powers for any positive integer n, but could not be expressed as the sum of fewer than nth powers, for any positive integer n (all numbers nonzero). He also noted the interesting fact that $59^4 + 158^4 = 133^4 + 134^4$.

Euler's conjecture was disproved for $n = 5$ in 1966 by L.J. Lander and T.R. Paiken who found the equation

$$144^5 = 27^5 + 84^5 + 110^5 + 133^5.$$

In 1986, Noam Elkies discovered the equation

$$20615673^4 = 2682440^4 + 15365639^4 + 18796760^4,$$

and subsequently showed that there were infinitely many fourth powers that were the sum of three nonzero fourth powers. The example with the smallest numbers was discovered in 1998 by Roger Frye:

$$422481^4 = 95800^4 + 217519^4 + 414560^4.$$

Fermat's Last Theorem has also entered into the popular culture. In a 1954 short story by Arthur Porges entitled *The Devil and Simon Flagg*, the protagonist makes a Faustian pact with the Devil for a solution of Fermat's result, with a somewhat whimsical result.