The rotating table

In 1975, commissioned by the Science Council of Canada, Professor John Coleman, head of mathematics at Queens University, along with two colleagues, G,D, Edwards and K.P. Beltzner, produced a background study on mathematics in Canada. At the time, he arranged a conference that brought together mathematicians, educational researchers, teachers and others involved in school education from across Canada. Substantive discussion by such a wide group was both unusual and very productive, and it was decided to have annual conferences. Thus came into being the Canadian Mathematics Education Study Group, which to this day continues to meet for about five days every May.

Not all the business of these conference occurs in the formal sessions. One year in the 1980s, the conference was held in Kingston, and a number of us had an evening stroll along the peaceful shore of Lake Ontario exchanging nice problems. Claude Gaulin, from Laval University, shared a problem he heard on a recent visit to Russia.

A circular table has in its surface four deep wells symmetrically placed at the vertices of a square. Within each well is placed a drinking tumbler which may be either upright or inverted. They are not all the same way up to begin with, and the task of the operator is to end up with them all upright or all inverted according to certain rules.

The operator cannot see into any of the wells. She proceeds by a number of moves. In each move, she may put her hands into two of the wells, determine the state of the tumblers, and for each of them, either leave it alone or turn it over. When she withdraws her hands, the table rotates and stops at random, at which point, another move can be made. When the task is complete, a bell will ring. Can the goal be achieved? If so, how?

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Our first reaction was that it may not be possible to finish. After all, since we have no control over where the table stops before each move, one of the wells may escape our attention and its tumbler cannot be reached. However, we soon realized that, while this indeed could occur, we still might be able to arrange to get the other tumblers into the same state.

To get on track for this problem, you need to realize that at each move there are two distinguishable possibilities: select two adjacent wells or two diagonally opposite wells. It is straightforward to see that in two moves, it can be arranged that three of the tumblers are in the same state. If this state agrees with that of the fourth, the bell will ring and we are done; otherwise, we know that three have the same state and the fourth has the other state.

I will let you take it from here. When you have solved this problem, see if you can succeed where the moves are made by a robot who cannot tell us the state of the tumblers when it puts its hands in, but can be instructed on how the tumblers can be oriented.