

MAT 212 MIDTERM. SPRING 2008.

This is a closed book test. Please, do not use textbooks, notes, calculators or any other external source of information. If you write with pencil, your work won't be regraded. Good luck!

- (1) It known, that the rate of which a rumor spreads in town is proportional both to the fraction of the population that knows it and to the fraction of the population that does not know it. Let $P(t)$ denote the proportion of the population aware of the rumor after time t .

(a) [4 points] Write a differential equation for $P(t)$,

(b) [4 points] Suppose, that the size of population is equal to 1000.

At 8 : 00am, 80 people knew the rumor and at noon 500 people knew it. Find when the rumor spreads to the 90% of the town.

- (2) [6 points] Find a linear differential equation with constant coefficients which has the function $y = \cos x + e^{-2x}$ as a particular solution
- (3) [6 points] Find the general solution to the differential equation

$$y'' + 2y' + y = 3e^{-x}\sqrt{x+1}$$

- (4) [6 points] Find the general solution to the differential equation

$$5(y''')^2 - 3y''y^{(4)} = 0$$

- (5) [6 points] Find the solution to the differential equation

$$y^2 + x^2y' = xyy',$$

satisfying the initial condition $y(1) = 1$

- (6) [6 points] Find the general solution to the differential equation

$$y''' - 3y'' + 2y' = 10 + 42e^{3x}$$

- (7) [6 points] Find the general solution to the equation

$$xy^2y' + y^3 = x \cos x$$

- (8) [6 points] Consider two balls attached to the identical springs. Assume that there are neither external forces nor damping (i.e. in vacuum). If the first ball oscillates twice as fast as the second, then how the masses of the balls are related?