

HOMEWORK ASSIGNMENT 9

Due Tuesday April 10, 2008 in class. This is the last class and no late homework will be accepted.

- (1) Find the equilibria for the following systems of differential equations and determined their type.

$$(a) \begin{cases} \dot{x} = x + y + 1 \\ \dot{y} = y + \sqrt{1 + 2x^2} \end{cases},$$

$$(b) \begin{cases} \dot{x} = x^2 - y \\ \dot{y} = \ln(1 - x + x^2) - \ln 3 \end{cases},$$

$$(c) \begin{cases} \dot{x} = \ln \frac{y^2 - y + 1}{3} \\ \dot{y} = x^2 - y^2 \end{cases},$$

$$(d) \begin{cases} \dot{x} = \ln(1 - y + y^2) \\ \dot{y} = 3 - \sqrt{x^2 + 8y} \end{cases},$$

$$(e) \begin{cases} \dot{x} = \sqrt{(x - y)^2 + 3} - 2 \\ \dot{y} = e^{y^2 - x} - e \end{cases},$$

- (2) Use polar coordinates to find limit cycles of the following systems.

Determine whether they are attracting, repelling or neither.

$$(a) \begin{cases} \dot{x} = y + x(1 - \sqrt{x^2 + y^2}) \\ \dot{y} = -x + y(1 - \sqrt{x^2 + y^2}) \end{cases},$$

$$(b) \begin{cases} \dot{x} = -3y + x(4 - x^2 - y^2) \\ \dot{y} = 3x + y(4 - x^2 - y^2) \end{cases},$$

$$(c) \begin{cases} \dot{x} = -y + x(\sqrt{x^2 + y^2} - 3 + \frac{2}{\sqrt{x^2 + y^2}}) \\ \dot{y} = x + y(\sqrt{x^2 + y^2} - 3 + \frac{2}{\sqrt{x^2 + y^2}}) \end{cases},$$

- (3) Consider the system

$$\begin{cases} \dot{x} = y \\ \dot{y} = -x - y^3 \end{cases},$$

Use the chain rule to calculate dV/dt for the following functions

$V(x, y)$:

$$(a) V(x, y) = x^2 + y^2, \quad (b) V(x, y) = x^2 + xy - y^2$$

$$(c) V(x, y) = 2x^2 + 3xy - y^2 \quad (d) V(x, y) = x^3 + y^3$$

- (4) Check that the function $V(x, y) = x^2 + y^2$ is a Lyapunov function for the following system and decide whether it has an asymptotically stable equilibrium point at the origin:

$$\begin{cases} \dot{x} = -x - 2y^2 \\ \dot{y} = 2xy - y^3 \end{cases},$$

- (5) Check that the function $V(x, y) = \frac{1}{2}x^4 + y^2$ is a Lyapunov function for the following system and decide whether it has an asymptotically stable equilibrium point at the origin:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -y^3 - x^2 \end{cases},$$