Homework Assignment 6

(Due Thursday March 13, 2008 in class)

(1) Compute the exponent e^A of a matrix A given by:

$$\text{a)}A = \begin{pmatrix} 4 & -3 \\ 6 & -7 \end{pmatrix}, \qquad \text{b)}A = \begin{pmatrix} -3 & 2 \\ -3 & 4 \end{pmatrix}$$

(2) Find the general solution of the following systems of differential equations *by computing the exponents of the corresponding matrices*:

$$\text{a)} \begin{cases} \dot{x} = 4x + 2y \\ \dot{y} = -3x - y \end{cases}, \qquad \text{b)} \begin{cases} \dot{x} = 3x - y \\ \dot{y} = 5x - y \end{cases}$$

(3) Find the solution of a system of differential equations with given initial conditions *by computing the exponent of the corresponding matrix*:

$$\begin{cases} \dot{x} = 4x + y \\ \dot{y} = -2x + y \end{cases}, \ x(0) = y(0) = 1$$

(4) The functions $x(t), y(t)$ satisfy a system of differential equations:

$$\begin{cases} \dot{x} = x + 2y \\ \dot{y} = 2x + y \end{cases}$$

with the initial conditions $x(0) = 1, y(0) = 2$. Find $x(10)$.

(5) Find the general solutions to the following systems of differential equations *without computing the exponents of the corresponding matrices*:

$$\text{a)} \begin{cases} \dot{x} = 2x + 3y \\ \dot{y} = 2x + y \end{cases}, \qquad \text{b)} \begin{cases} \dot{x} = -3x + 4y \\ \dot{y} = 6x - 5y \end{cases}$$

(6) Given the system of differential equations:

$$\begin{cases} \dot{x} = 9x + 5y \\ \dot{y} = -6x - 2y \end{cases}$$

and the initial conditions $x(0) = 1$, $y(0) = 0$, find $x(t)$. (*Do NOT compute the exponent of the corresponding matrix*)

(7) Find the general solutions to the following systems of differential equations *without computing the exponents of the corresponding matrices*:

$$\text{a)} \begin{cases} \dot{x} = 7x + y \\ \dot{y} = -4x + 3y \end{cases}, \qquad \text{b)} \begin{cases} \dot{x} = x - 4y \\ \dot{y} = 4x + 9y \end{cases}$$

(8) Solve the initial value problem for the following systems of differential equations:

(a) $\begin{cases} \dot{x} = x - 3y \\ \dot{y} = 3x + 7y \end{cases}$, $x(0) = y(0) = 1$

(b) $\begin{cases} \dot{x} = 4x + y \\ \dot{y} = 6x - y \end{cases}$, $x(0) = y(0) = 1$