Homework Assignment 4

(Due Thursday February 14, 2008 in class)

(1) Solve the following differential equations:

(a) $y'' + y = \sin^2 x$,

(b) $9y'' - 6y' + y = (xe^x)^{1/3}$,

(c) $y'' - 2y' + y = e^x \ln x$,

(d) $y'' + 3y' + 2y = \frac{1}{e^x+1}$,

(2) Find the general solution of the following differential equations:

(a) $y^{(IV)} + 5y'' + 4y = 0$,

(b) $y''' + 7y'' + 16y' + 12y = 0$,

(c) $y^{(IV)} + 2y''' + 10y'' = 0$,

(d) $y^{(IV)}) - 4y''' + 6y'' - 4y' + y = 0$,

(e) $y''' - y' = \sin x$,

(f) $y^{(IV)} - y'' = 4xe^x$,

(3) Find the solution to the differential equation satisfying corresponding initial conditions:

(a) $5y'' - 10y' + 5y = 60x^{-5}e^x$, $y(\ln 2) = y'(\ln 2) = 0$,

(b) $y''' - 3y' - 2y = e^{2x}$, $y(0) = 0$, $y'(0) = -3$, $y''(0) = 3$,

(4) The equation of the form

$$x^2y'' + axy' + by = f(x)$$

is called *Euler's equation.* It can be reduced to a second order linear differential equation by the substitution $x = e^t$. Use this substitution to solve the following differential equations:

(a) $x^2y'' - 4xy' + 6y = 0$,

(b) $x^2y'' - xy' + y = 2t$,

(c) $x^3y'' - 2xy = 6\ln x$.