MAT 1060: Partial Differential Equations I
Assignment 1, September 16 2009

Read Chapter 1 and Chapter 2 up to p. 43. Look through p. 626-628 of Appendix C.

Please hand in to Ehsan in class on Wednesday, September 23.

- Chapter 1 (p. 12-13): Problems 1, 4.
- Chapter 2 (p. 85): Problems 1, 2, 3, 4.

including the additional problem:

- *(The Laplacian in polar coordinates.)*

  Let u be a smooth real-valued function in two variables, and define

  $$v(r, \theta) = u(r \cos \theta, r \sin \theta) \, ,$$

  i.e., v is obtained from u by transforming from Cartesian into polar coordinates. Express the Laplacian Δu in terms of derivatives of v.
  Hint: First compute $Dv(r, \theta)$ in terms of u.

- *Harmonic functions vs. holomorphic functions*

  Let U be an open set in the complex plane, and consider a holomorphic function f on U.

  (a) Interpret f as a vector-valued function of two variables, by writing $z = x + iy$ and $f(z) = u(x, y) + iv(x, y)$. Write down the Cauchy-Riemann differential equations for u and v.

  (b) Show that u and v are harmonic.

  (c) Given a harmonic function u on an open set $U \subset \mathbb{R}^2$, prove that there exists a harmonic function v such that $u + iv$ is holomorphic.
  Remark: v is called the *conjugate hamonic function* to u. It is unique up to adding a constant.