MAT137 - Calculus with proofs

- Test 3 opens on February 5.

Unit 9: Integration methods

- Substitution or Chain Rule
  (Videos 9.1, 9.3; Supplementary: 9.2)
- Parts or Product Rule
  (Video: 9.4; Supplementary: 9.5, 9.6)
- TODAY: Products of trig functions
  (Video: 9.7; Supplementary: 9.8, 9.9)
- NEXT: Rational functions
  (**Video: 9.10**; Supplementary: 9.11, 9.12)

Practice: Integrals with trigonometric functions

Compute the following antiderivatives. (Once you get them to a form from where you see a path to finish them, even if long, you may stop.)

1. $\int \sin^{10} x \cos x \; dx$
2. $\int \sin^{10} x \cos^{7} x \, dx$
3. $\int e^{\cos x} \cos x \sin^{3} x \, dx$
4. $\int \cos^{2} x \, dx$
5. $\int \cos^{4} x \, dx$
6. $\int \csc x \, dx$

Useful trig identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $\square$, then use the substitution $u = \tan x$.
- If $\square$, then use the substitution $u = \sec x$.

Hint: You will need

- $\dfrac{d}{dx}[\tan x] = \ldots$
- $\dfrac{d}{dx}[\sec x] = \ldots$
- The trig identity involving sec and tan

The error function

The following function is tabulated: $E(x) = \int_0^x e^{-t^2} dt$.

Write the following quantities in terms of E:

1. $\int_1^2 e^{-t^2} dt$
2. $\int_0^x t^2 e^{-t^2} dt$
3. $\int_0^x e^{-2t^2} dt$
4. $\int_0^1 e^{-t^2+6t} dt$
5. $\int_{x_1}^{x_2} e^{-\frac{(t-\mu)^2}{\sigma^2}} dt$
6. $\int_1^2 \frac{e^{-t}}{\sqrt{t}} dt$