MAT137 - Calculus with proofs

- Assignment #6 due on January 28.
- Test 3 on February 5.

- Today: Antiderivatives and functions defined as integrals.

- MONDAY: The Fundamental Theorem of Calculus - Part 1 (**Videos 8.3, 8.4**)

The most misunderstood antiderivative

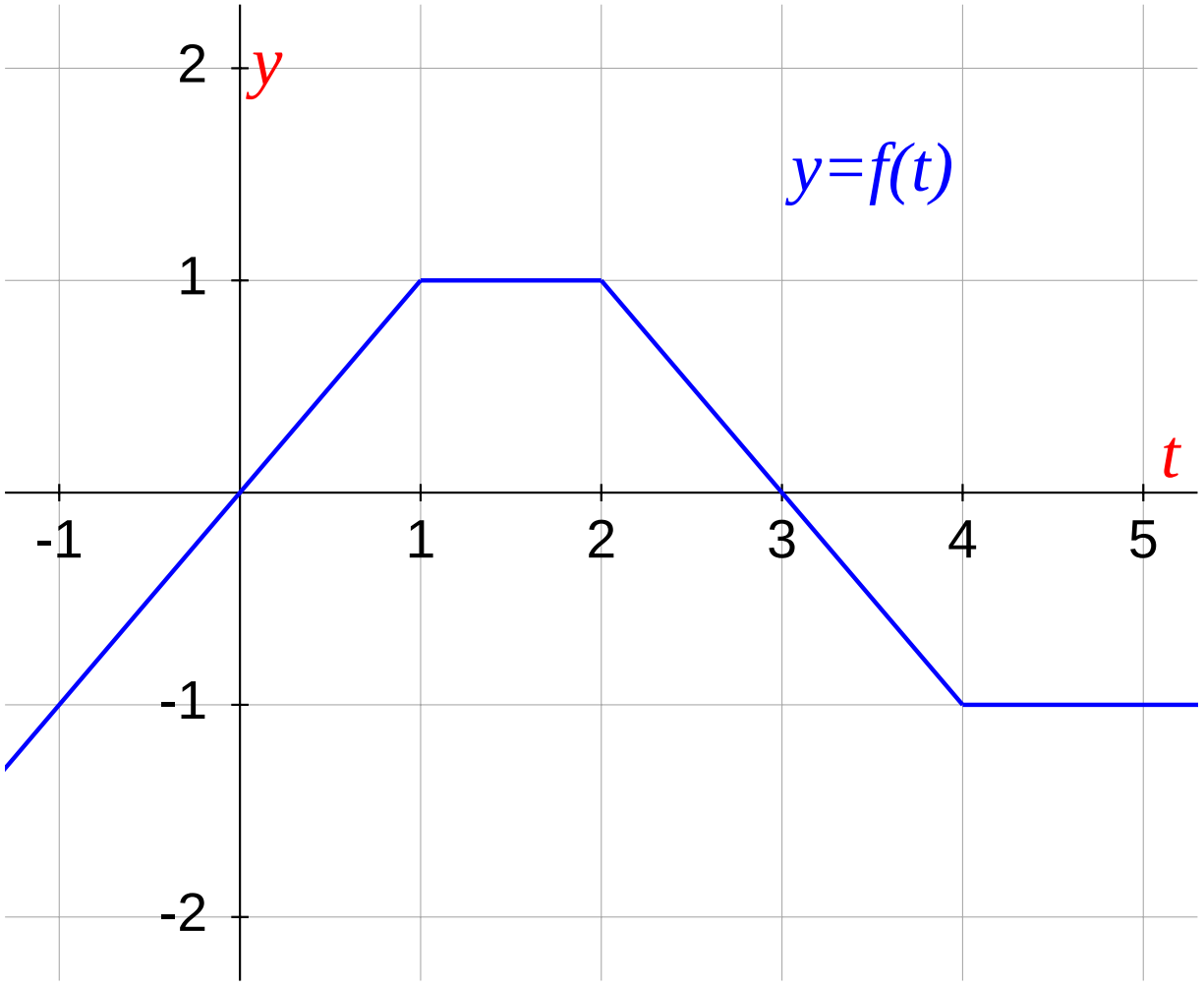
1. Find the *domain* and the derivative of $F_1(x) = \ln x$
2. Find the *domain* and the derivative of $F_2(x) = \ln(-x)$
3. Find the *domain* and the derivative of $F_3(x) = \ln|x|$
   Suggestion: Break the domain into two pieces.
4. Based on your answers, what is $\displaystyle\int \frac{1}{x}\,dx$?
5. Find the *domain* and the derivative of $F_4(x) = \ln|2x|$
   Why doesn't this contradict your answer to 4 ?

Functions defined by integrals

Which ones of these are valid ways to define functions?

1. $F(x) = \displaystyle\int_0^x \frac{t}{1+t^8}\,dt$
2. $F(x) = \displaystyle\int_0^x \frac{x}{1+x^8}\,dx$
3. $F(x) = \displaystyle\int_0^x \frac{x}{1+t^8}\,dt$
4. $F(x) = \displaystyle\int_0^{x^2} \frac{t}{1+t^8}\,dt$
5. $F(x) = \displaystyle\int_{\sin x}^{e^x} \frac{t}{1+t^8}\,dt$
6. $F(x) = \displaystyle\int_0^3 \frac{t}{1+x^2+t^8}\,dt$
7. $F(x) = x\displaystyle\int_{\sin x}^{e^x} \frac{t}{1+x^2+t^8}\,dt$
8. $F(x) = t\displaystyle\int_{\sin x}^{e^x} \frac{t}{1+x^2+t^8}\,dt$

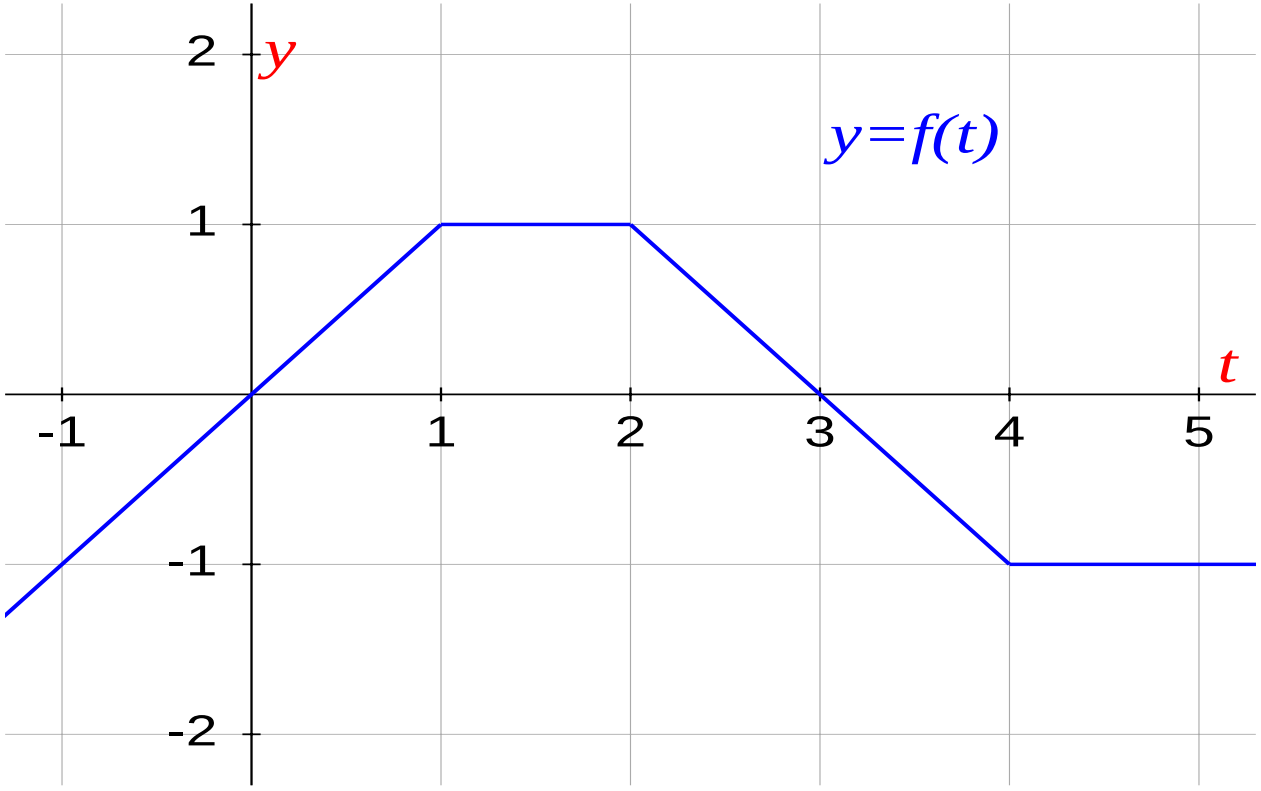
Towards FTC



Compute:

1. $\int_0^1 f(t)dt$
2. $\int_0^2 f(t)dt$
3. $\int_0^3 f(t)dt$
4. $\int_0^4 f(t)dt$
5. $\int_0^5 f(t)dt$

Towards FTC (continued)



Call $F(x) = \int_0^x f(t)dt$. This is a new function.

- Sketch the graph of $y = F(x)$.
- Using the graph you just sketched, sketch the graph of $y = F'(x)$.

Compute these antiderivatives by guess 'n check

1. $\int x^5 dx$
2. $\int \left(3x^8 - 18x^5 + 1\right) dx$
3. $\int \sqrt[3]{x}\, dx$
4. $\int \frac{1}{x^9}\, dx$
5. $\int \sqrt{x}\left(x^2 + 5\right) dx$
6. $\int \frac{1}{e^{2x}}\, dx$
7. $\int \sin(3x)\, dx$
8. $\int \cos(3x + 2)\, dx$
9. $\int \sec^2 x\, dx$
10. $\int \sec x \tan x\, dx$
11. $\int \frac{1}{x}\, dx$
12. $\int \frac{1}{x + 3}\, dx$