MAT137 - Calculus with proofs

- Assignment #6 due on January 28.

- Today: Suprema and infima.

- FRIDAY: Definition of integral **(Videos 7.5, 7.6)**

Warm up: suprema and infima

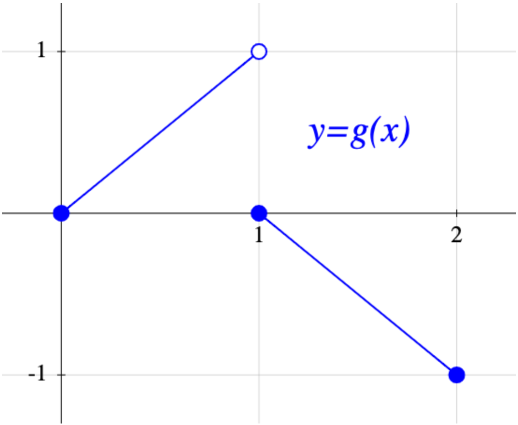
Find the supremum, infimum, maximum, and minimum of the following sets (if they exist):

1. $[-1, 5)$
2. $(-\infty, 6] \cup (8, 9)$
3. $\{2, 3, 4\}$
4. $\left\{ \dfrac{1}{n} \ : \ n \in \mathbb{Z},\ n > 0 \right\}$
5. $\left\{ \dfrac{(-1)^n}{n} \ : \ n \in \mathbb{Z},\ n > 0 \right\}$
6. $\{2^n \ : \ n \in \mathbb{Z}\}$

Suprema from a graph

Calculate, for the function g on the interval $[0.5, 1.5]$:

1. supremum 2. infimum 3. maximum 4. minimum



Empty set

1. Does $\emptyset$ have an upper bound ?
2. Does $\emptyset$ have a supremum?
3. Does $\emptyset$ have a maximum?
4. Is $\emptyset$ bounded above?

Equivalent definitions of supremum

Assume S is an upper bound of the set A.
Which of the following is equivalent to "S is the supremum of A"?

1. If R is an upper bound of A, then $S \leq R$.
2. $\forall R \geq S$, R is an upper bound of A.
3. $\forall R \leq S$, R is not an upper bound of A.
4. $\forall R < S$, R is not an upper bound of A.
5. $\forall R < S$, $\exists x \in A$ such that $R < x$.
6. $\forall R < S$, $\exists x \in A$ such that $R \leq x$.
7. $\forall R < S$, $\exists x \in A$ such that $R < x \leq S$.
8. $\forall R < S$, $\exists x \in A$ such that $R < x < S$.
9. $\forall \varepsilon > 0$, $\exists x \in A$ such that $S - \varepsilon < x$.
10. $\forall \varepsilon > 0$, $\exists x \in A$ such that $S - \varepsilon < x \leq S$.