- Assignment #6 due on January 28.

- Today: Sums and sigma notation.

- WED: Suprema & infima   (**Videos 7.3, 7.4**)
- FRI: The definition of integral   (Videos 7.5, 7.6)

Reminders:

- Watch videos before class
- Download slides before class (no need to read them)

Compute

1. $\displaystyle\sum_{i=2}^{4}(2i+1)$

2. $\displaystyle\sum_{i=2}^{4}2i+1$

3. $\displaystyle\sum_{j=2}^{4}(2i+1)$

Write these sums with Σ notation

1. $1^5 + 2^5 + 3^5 + 4^5 + \ldots + 100^5$

2. $\dfrac{2}{4^2} + \dfrac{2}{5^2} + \dfrac{2}{6^2} + \dfrac{2}{7^2} + \ldots + \dfrac{2}{N^2}$

3. $\cos 0 - \cos 1 + \cos 2 - \cos 3 + \ldots \pm \cos(N+1)$

4. $\dfrac{1}{0!} + \dfrac{1}{2!} + \dfrac{1}{4!} + \dfrac{1}{6!} + \ldots + \dfrac{1}{(2N)!}$

5. $\dfrac{1}{1!} - \dfrac{1}{3!} + \dfrac{1}{5!} - \dfrac{1}{7!} + \ldots + \dfrac{1}{81!}$

6. $\dfrac{2x^3}{4!} + \dfrac{3x^4}{5!} + \dfrac{4x^5}{6!} + \ldots + \dfrac{999x^{1000}}{1001!}$

Telescopic sum

- Calculate the exact value of

$$\sum_{i=1}^{137} \left[\frac{1}{i} - \frac{1}{i+1} \right]$$

  Hint: Write down the first few terms.

- Calculate the exact value of

$$\sum_{i=1}^{10,000} \frac{1}{i(i+1)}$$

Double sums

Compute:

1. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{N}1$

2. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i}1$

3. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i}i$

4. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i}k$

5. $\displaystyle\sum_{i=1}^{N}\sum_{k=1}^{i}(ik)$

Useful formulas:

$$\sum_{j=1}^{N}j = \frac{N(N+1)}{2}, \quad \sum_{j=1}^{N}j^2 = \frac{N(N+1)(2N+1)}{6}, \quad \sum_{j=1}^{N}j^3 = \frac{N^2(N+1)^2}{4}$$