MAT137 - Calculus with proofs

- We have different office hours during Reading Week.
- Assignment #4 due on November 26

- TODAY: Exponentials and logarithms

- Monday after Reading Week: Inverse trigonometric functions **(Watch videos 4.12, 4.13, 4.14)**

Compute the derivative of the following functions:

1. $f(x) = e^{\sin x + \cos x} \ln x$

2. $f(x) = \pi^{\tan x}$

3. $f(x) = \ln \left[e^x + \ln \ln \ln x \right]$

Let $F(x) = \ln |x|$.
Compute its derivative.

1. $F'(x) = \dfrac{1}{x}$

2. $F'(x) = \dfrac{1}{|x|}$

3. F is not differentiable

Logarithmic differentiation

Calculate the derivative of

$$g(x) = x^{\tan x}.$$

More logarithmic differentiation

Calculate the derivative of

$$f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}.$$

What is wrong with this answer?

$$\ln f(x) = (\cos x)\ln(\sin x) + (\sin x)(\ln \cos x)$$

$$\frac{d}{dx}[\ln f(x)] = \frac{d}{dx}[(\cos x)\ln(\sin x)] + \frac{d}{dx}[(\sin x)(\ln \cos x)]$$

$$\frac{f'(x)}{f(x)} = -(\sin x)\ln(\sin x) + (\cos x)\frac{\cos x}{\sin x}$$

$$+ (\cos x)\ln(\cos x) + (\sin x)\frac{-\sin x}{\cos x}$$

$$f'(x) = f(x)\left[-(\sin x)\ln(\sin x) + (\cos x)\ln(\cos x) + \frac{\cos^2 x}{\sin x} - \frac{\sin^2 x}{\cos x}\right]$$

Cue boss music

Calculate the derivative of

$$h(x) = \sqrt[3]{\frac{(\sin^6 x)\,\sqrt{x^7 + 6x + 2}}{3^x\,(x^{10} + 2x)^{10}}}$$

Use the basic differentiation rules, as well as

$$\frac{d}{dx} \sin x = \cos x, \qquad \frac{d}{dx} \cos x = -\sin x,$$

to quickly obtain and prove formulas for the derivatives of tan, cot, sec, and csc.