MAT137 - Calculus with proofs

- Test 1 is open from 3pm today to 3pm Saturday
- Assignment #3 due on November 5

- TODAY: Differentiation rules

- MONDAY : Proof of differentiation rules

  (**Videos 3.6, 3.7, 3.9)**

- WEDNESDAY: Chain Rule (Videos 3.10, 3.11)

Differentiable functions

Let $a \in \mathbb{R}$.
Let f be a function with domain $\mathbb{R}$.
Assume f is differentiable everywhere.
What can we conclude?

1. $f(a)$ is defined.
2. $\lim_{x \to a} f(x)$ exists.
3. f is continuous at a.
4. $f'(a)$ exists.
5. $\lim_{x \to a} f'(x)$ exists.
6. f' is continuous at a.

Computations: Basic differentiation rules

Compute the derivative of the following functions:

1. $f(x) = x^{100} - 3x^9 - 2$
2. $f(x) = \sqrt[3]{x} + 6$
3. $f(x) = \dfrac{4}{x^4}$
4. $f(x) = \sqrt{x}\,(1 + 2x)$
5. $f(x) = \dfrac{x^6 + 1}{x^3}$
6. $f(x) = \dfrac{x^2 - 2}{x^2 + 2}$

Higher order derivatives

Let $g(x) = \dfrac{1}{x^3}$.

- Calculate the first few derivatives.
- Make a conjecture for a formula for the n-th derivative $g^{(n)}(x)$.
- Prove it by induction.

Estimations - 1

Let f be a continuous function with domain $\mathbb{R}$.
We know $f(4) = 3$ and $f(4.2) = 2.2$.
Based only on this, give your best estimate for $f(4.1)$.

Estimations - 2

Let f be a continuous function with domain $\mathbb{R}$.
We know $f(4) = 3$ and $f(4.1) = 4$.
Based only on this, give your best estimate for $f'(4)$.

Estimations - 3

Let f be a continuous function with domain $\mathbb{R}$.
We know $f(4) = 3$ and $f'(4) = 0.5$.
Based only on this, give your best estimate for $f(4.1)$.

Estimations – 4

Without using a calculator, estimate $\sqrt[20]{1.01}$.

Hint: You know the value of $f(x) = \sqrt[20]{x}$ and its derivative at one point very close to 1.01. Use the tangent line at that point as an approximation.