MAT137 - Calculus with proofs

- Test 1: Friday 3pm to Saturday 3pm
- Assignment #3 due on November 5

- TODAY: EVT and IVT

- WEDNESDAY: Derivatives! **(Videos 3.1, 3.2, 3.3)**

Existence of solutions

Prove that the equation

$$x^4 - 2x = 100$$

has at least two solutions.

Can this be proven?

1. Prove that there exists a time of the day when the hour hand and the minute hand of a clock form an angle of exactly 23 degrees.

2. During a Raptors basketball game, at half time the Raptors have 52 points. Prove that at some point they had exactly 26 points.

3. Prove that at some point during Alfonso's life, his height in centimetres was exactly equal to 10 times his weight in kilograms. Some data:
   - His height at birth: 47 cm
   - His weight at birth: 3.2 kg
   - His height today: 172 cm

Definition of maximum

Let f be a function with domain I.
Which one (or ones) of the following is (or are) a definition of "f has a maximum on I"?

1. $\forall x \in I, \exists C \in \mathbb{R}$ s.t. $f(x) \leq C$
2. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) \leq C$
3. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) < C$

Extrema

In each of the following cases, does the function f have a maximum and a minimum on the interval I?

1. $f(x) = x^2, \quad I = (-1, 1).$
2. $f(x) = \dfrac{\sin x}{x} - \cos x + 3, \quad I = [2, 6]$
3. $f(x) = \dfrac{\sin x}{x} - \cos x + 3, \quad I = (0, 8]$
4. $f(x) = \left[\sin^2 x\right] \left[\sin^2(\pi x)\right], \quad I = \mathbb{R}$