MAT137 - Calculus with proofs

- TODAY: Continuity

- MONDAY is a holiday (no class)

- WEDNESDAY: More Continuity
  - **Required videos 2.16, 2.17**
  - Supplementary video: 2.18

Undefined function

Let $a \in \mathbb{R}$ and let f be a function. Assume $f(a)$ is undefined.

What can we conclude?
1. $\lim\limits_{x \to a} f(x)$ exist
2. $\lim\limits_{x \to a} f(x)$ doesn't exist.
3. No conclusion. $\lim\limits_{x \to a} f(x)$ may or may not exist.

What else can we conclude?
4. f is continuous at a.
5. f is not continuous at a.
6. No conclusion. f may or may not be continuous at a.

The definition of continuity

Let f be a function with domain $\mathbb{R}$. Let $a \in \mathbb{R}$.

Which statements are equivalent to "f is continuous at a"?

1. $\lim\limits_{x \to a} f(x)$ exists

2. $\lim\limits_{x \to a} f(x)$ exists and $f(a)$ is defined.

3. $\lim\limits_{x \to a} f(x) = f(a)$

4. $\forall \varepsilon > 0, \ \exists \delta > 0, \ \forall x \in \mathbb{R}, \ 0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon$

5. $\forall \varepsilon > 0, \ \exists \delta > 0, \ \forall x \in \mathbb{R}, \ 0 < |x - a| < \delta \implies |f(x) - f(a)| < \varepsilon$

6. $\forall \varepsilon > 0, \ \exists \delta > 0, \ \forall x \in \mathbb{R}, \qquad |x - a| < \delta \implies |f(x) - f(a)| < \varepsilon$

A new function

- Let $x, y \in \mathbb{R}$. What does the following expression calculate? Prove it.
$$f(x, y) = \frac{x + y + |x - y|}{2}$$

Suggestion: If you don't know how to start, try some sample values of x and y.

- Write a similar expression to compute $\min\{x, y\}$.

More continuous functions

We want to prove the following theorem

Theorem

IF f and g are continuous functions
THEN $h(x) = \max\{f(x), g(x)\}$ is also a continuous function.

You are allowed to use all results that we already know. What is the fastest way to prove this?

Hint: There is a way to prove this quickly without writing any epsilons.

Existence

Write the definition of these statements:

1. $\lim\limits_{x \to -\infty} f(x) = L$
2. $\lim\limits_{x \to -\infty} f(x)$ does not exist

Write the negation of these statements:

1. If Justin Trudeau has a brother, then he also has a sister.
2. If a student in this class has a brother, then they also have a sister.