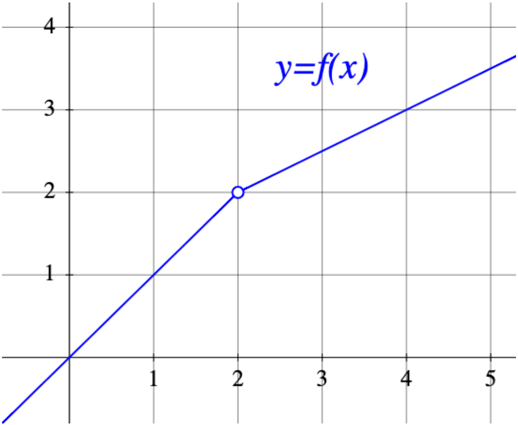
MAT137 - Calculus with proofs

- Assignment #1 due TOMORROW

- TODAY: The formal definition of limit

- FRIDAY: Proofs with the definition of limit:
  - **Required videos: 2.7, 2.8**
  - Supplementary video: 2.9

δ from a graph



Find *all* values of $\delta > 0$ that satisfy

$$0 < |x - 2| < \delta \implies |f(x) - 2| < 0.5$$

Warm-up

Write down the formal definition of

$$\lim_{x \to a} f(x) = L.$$

Side limits

Recall

Let $L, a \in \mathbb{R}$.
Let f be a function defined at least on an interval around a, except possibly at a.

$$\lim_{x \to a} f(x) = L$$

means

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t.} \qquad 0 < |x-a| < \delta \implies |f(x)-L| < \varepsilon.$$

Write, instead, the formal definition of

$$\lim_{x \to a^+} f(x) = L, \qquad \text{and} \qquad \lim_{x \to a^-} f(x) = L.$$

Infinite limits

Let $a \in \mathbb{R}$. Let f be a function defined at least on an interval around a, except possibly at a.

Write a formal definition for $\lim_{x \to a} f(x) = \infty$.

Infinite limits

Which ones are (equivalent to) the definition of $\lim_{x \to a} f(x) = \infty$?

1. $\forall \varepsilon > 0, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies |f(x) - \infty| < \varepsilon$
2. $\forall M > 0, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies |f(x) - L| > M$
3. $\forall \delta > 0, \exists M > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
4. $\forall \varepsilon > 0, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > \varepsilon$
5. $\forall M > 0, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$
6. $\forall M \in \mathbb{R}, \exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies f(x) > M$