MAT137 - Calculus with proofs

- Assignment 10 due on April 8
- Test 5 opens on April 22

- Monday: Constructing new power series
- Wednesday: Applications! **(Videos 14.12, 14.14)**

- Please fill out course evaluations

Taylor series gymnastics

Write the following functions as power series centered at 0. Write them first with sigma notation, and then write out the first few terms. Indicate the domain where each expansion is valid.

1. $f(x) = e^{-x}$
2. $f(x) = x^2 \cos x$
3. $f(x) = \dfrac{1}{1+x}$
4. $f(x) = \dfrac{1}{1-x^2}$
5. $f(x) = \dfrac{x}{3+2x}$
6. $f(x) = \sin\left(2x^3\right)$
7. $f(x) = \dfrac{e^x + e^{-x}}{2}$
8. $f(x) = \ln \dfrac{1+x}{1-x}$

Note: You do not need to take any derivatives. You can reduce them all to other Maclaurin series you know.

arctan

1. Write the Maclaurin series for $G(x) = \arctan x$

   Hint: Compute the first derivative. Then use the geometric series. Then integrate.

2. What is $G^{(137)}(0)$?

3. Compute

$$A = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\,3^n}$$

arcsin

Recall (from last week): For $|x| < 1$:

$$f(x) = \frac{1}{\sqrt{1+x}} = 1 + \sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!!}{(2n)!!} x^n$$

$$= 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \dots$$

4. Write $h(x) = \arcsin x$ as a power series centered at 0. Write it with sigma notation, and also write out the first few terms.

   Hint: Compute $h'(x)$ and relate it to $f(x)$. Then integrate.

5. What is $h^{(137)}(0)$?