## MAT137 - Calculus with proofs

- Assignment 10 due on April 8
- Test 5 opens on April 22
- Today: Taylor series
- Wednesday: Analytic functions
(Videos 14.5, 14.6)
(Videos 14.7, 14.8)
- Friday: no class (Good Friday)
- Next week: Profit!


## Competition!

- Do you prefer cats or dogs? You MUST choose one. Now you are in the $C$-team or the $D$-team.
- Copy only one polynomial ( $C$ or $D$ ):

$$
\begin{aligned}
& C(x)=-\frac{293}{8}+29 x+\frac{13}{4} x^{2}-3 x^{3}+\frac{3}{8} x^{4} \\
& D(x)=29+8(x-3)-\frac{7}{2}(x-3)^{2}+\frac{9}{6}(x-3)^{3}+\frac{9}{24}(x-3)^{4}
\end{aligned}
$$

- I will ask you questions.

Answer only about your polynomial ( $C$ or $D$ ). No calculators!

## Competition!

$$
\begin{aligned}
& C(x)=-\frac{293}{8}+29 x+\frac{13}{4} x^{2}-3 x^{3}+\frac{3}{8} x^{4} \\
& D(x)=29+8(x-3)-\frac{7}{2}(x-3)^{2}+\frac{9}{6}(x-3)^{3}+\frac{9}{24}(x-3)^{4}
\end{aligned}
$$

C-team compute...

$$
\begin{array}{ll}
\text { 0. } & C(3) \\
\text { 1. } & C^{\prime}(3) \\
\text { 2. } & C^{\prime \prime}(3) \\
\text { 3. } & C^{\prime \prime \prime}(3) \\
\text { 4. } & C^{(4)}(3)
\end{array}
$$

D-team compute...
0. $D(3)$

1. $D^{\prime}(3)$
2. $D^{\prime \prime}(3)$
3. $D^{\prime \prime \prime}(3)$
4. $D^{(4)}(3)$

Simplify your answers (write them as rational numbers) No calculators!

## I spy a polynomial with my little eye

I'm thinking of a cubic polynomial $P$. It satisfies

$$
P(1)=8, \quad P^{\prime}(1)=-\pi, \quad P^{\prime \prime}(1)=4, \quad P^{\prime \prime \prime}(1)=\sqrt{7}
$$

What is $P(x)$ ?

## A new Maclaurin series

Let $f(x)=\frac{1}{\sqrt{1+x}}$.

1. Find a formula for its derivatives $f^{(n)}(x)$.

Note: Leave the coefficients factored (do not multiply them).
You may find the "double factorial" notation useful:

$$
7!!=7 \cdot 5 \cdot 3 \cdot 1, \quad 8!!=8 \cdot 6 \cdot 4 \cdot 2
$$

2. Write its Maclaurin series at 0 . Call it $S(x)$.

Use sigma notation, and write out the first few terms explicitly as well.

Note: It may be useful to separate the 0-th order term and not include it in the "sigma".
3. What is the radius of convergence of series $S(x)$ ?

## arcsin

You may use without proof that for every $x \in(-1,1)$,
$f(x)=\frac{1}{\sqrt{1+x}}=S(x)$, which you just computed.
4. Write $h(x)=\arcsin x$ as a power series centered at 0 .

Hint: Compute $h^{\prime}(x)$ and relate it to $f(x)$. Then integrate.
5. What is $h^{(137)}(0)$ ?

