# MAT137 - Calculus with proofs

- Assignment 10 due on April 8
- Test 5 opens on April 22

- Today: Taylor series (Videos 14.5, 14.6)
- Wednesday: Analytic functions (Videos 14.7, 14.8)
- Friday: no class (Good Friday)
- Next week: Profit!

# Competition!

- Do you prefer cats or dogs? You MUST choose one. Now you are in the *C*-team or the *D*-team.
- Copy only one polynomial (C or D):

$$C(x) = -\frac{293}{8} + 29x + \frac{13}{4}x^2 - 3x^3 + \frac{3}{8}x^4$$
  
$$D(x) = 29 + 8(x-3) - \frac{7}{2}(x-3)^2 + \frac{9}{6}(x-3)^3 + \frac{9}{24}(x-3)^4$$

 I will ask you questions. Answer only about your polynomial (C or D).
No calculators!

# Competition!

$$C(x) = -\frac{293}{8} + 29x + \frac{13}{4}x^2 - 3x^3 + \frac{3}{8}x^4$$
  
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C-team compute...

*D*-team compute...

0.	C(3)	0.	D(3)
1.	<i>C</i> ′(3)	1.	D'(3)
2.	<i>C</i> ″(3)	2.	D"(3)
3.	<i>C'''</i> (3)	3.	D'''(3)
4.	$C^{(4)}(3)$	4.	$D^{(4)}(3)$

Simplify your answers (write them as rational numbers) **No calculators!** 

### I spy a polynomial with my little eye

I'm thinking of a cubic polynomial P. It satisfies

$$P(1)=8, \quad P'(1)=-\pi, \quad P''(1)=4, \quad P'''(1)=\sqrt{7}$$

What is P(x)?

#### A new Maclaurin series

Let 
$$f(x) = \frac{1}{\sqrt{1+x}}$$

1. Find a formula for its derivatives  $f^{(n)}(x)$ .

*Note:* Leave the coefficients factored (do not multiply them). You may find the "double factorial" notation useful:

 $7!! = 7 \cdot 5 \cdot 3 \cdot 1, \qquad 8!! = 8 \cdot 6 \cdot 4 \cdot 2$ 

2. Write its Maclaurin series at 0. Call it S(x). Use sigma notation, and write out the first few terms explicitly as well.

Note: It may be useful to separate the 0-th order term and not include it in the "sigma".

3. What is the radius of convergence of series S(x)?

#### arcsin

You may use without proof that for every  $x \in (-1, 1)$ ,  $f(x) = \frac{1}{\sqrt{1+x}} = S(x)$ , which you just computed.

4. Write  $h(x) = \arcsin x$  as a power series centered at 0.

*Hint:* Compute h'(x) and relate it to f(x). Then integrate.

5. What is  $h^{(137)}(0)$ ?