MAT137 - Calculus with proofs

- Assignment 10 due on April 8
- Test 5 opens on April 22

- Today: Taylor polynomials **(Videos 14.3, 14.4)**
- Monday: Taylor series (Videos 14.5, 14.6)
- Wednesday: Analytic functions (Videos 14.7, 14.8)

Warm up

Write down the (various equivalent) definitions of Taylor polynomial you have learned so far.

Taylor polynomials of a polynomial

Let $f(x) = x^3$.
Let P_n be the n-th Taylor polynomial for f at 0.

1. Find P_3.
   Verify it using the 1st and 2nd definitions.

2. Find P_2.

More Taylor polynomials of a polynomial

Let $f(x) = x^3$.
Let Q_n be the n-th Taylor polynomial for f at 1.

3. Find Q_3.
4. Find Q_2.

A polynomial given its derivatives

1. Consider the polynomial $P(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3$. Find values of the coefficients that satisfy

$$P(0) = 1, \quad P'(0) = 5, \quad P''(0) = 3, \quad P'''(0) = -7$$

2. Find *all* polynomials P (of any degree) that satisfy

$$P(0) = 1, \quad P'(0) = 5, \quad P''(0) = 3, \quad P'''(0) = -7$$

3. Find a polynomial P of smallest possible degree that satisfies

$$P(0) = A, \quad P'(0) = B, \quad P''(0) = C, \quad P'''(0) = D$$

A polynomial given its derivatives

4. Let f be a C^4 function at 0. Construct a polynomial P that satisfies

$$P(0) = f(0)$$
$$P'(0) = f'(0)$$
$$P''(0) = f''(0)$$
$$P^{(3)}(0) = f^{(3)}(0)$$
$$P^{(4)}(0) = f^{(4)}(0)$$