MAT137 - Calculus with proofs

- Assignment 8 due on March 4
- Test 4 opens on March 12

- Today: Improper integrals

- Quick ways to determine whether an improper integral is convergent without computing its value:
  - Wednesday: BCT **(Videos 12.7, 12.8)**
  - Friday: LCT (Videos 12.9, 12.10)

Recall the definitions

1. **Type-1 improper integrals.** Let f be a bounded, continuous function on $[c, \infty)$. How do we define this improper integral?

$$\int_c^\infty f(x)dx$$

2. **Type-2 improper integrals.** Let f be an unbounded continuous function on $(a, b]$, possibly with vertical asymptote $x = a$. How do we define this improper integral?

$$\int_a^b f(x)dx$$

Computation

Calculate, using the definition of improper integral

$$\int_{1}^{\infty} \frac{1}{x^2 + x} dx$$

Hint: $\dfrac{1}{x^2 + x} = \dfrac{(x+1) - (x)}{x(x+1)}$

The most important improper integrals

Use the definition of improper integral to determine for which values of $p \in \mathbb{R}$ each of the following improper integrals converges.

1. $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$

2. $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$

3. $\displaystyle\int_0^\infty \frac{1}{x^p}\,dx$

Positive functions

Let f be continuous on $[a, \infty)$. Let $A = \int_a^\infty f(x)dx$

$$\text{Then } A \text{ may be } \begin{cases} \text{convergent (a number)} \\ \text{divergent} \begin{cases} \text{to } \infty \\ \text{to } -\infty \\ \text{``oscillating''} \end{cases} \end{cases}$$

1. Assume $\forall x \geq a, f(x) \geq 0$.

Which of the four options are still possible?

2. Assume $\exists M \geq a,$ s.t. $\forall x \geq M, f(x) \geq 0$.

Which of the four options are still possible?