MAT137 - Calculus with proofs

- Assignment 8 due on March 4
- Test 4 opens on March 12

- Today: The Big Theorem

- Monday: Improper integrals
  - **Watch Videos 12.1, 12.4, 12.5**
  - Supplementary videos: 12.2, 12.3, 12.6

Calculations

1. $$\lim_{n\to\infty} \frac{n! + 2e^n}{3n! + 4e^n}$$

2. $$\lim_{n\to\infty} \frac{2^n + (2n)^2}{2^{n+1} + n^2}$$

3. $$\lim_{n\to\infty} \frac{5n^5 + 5^n + 5n!}{n^n}$$

True or False – The Big Theorem

Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be positive sequences.

1. IF $a_n << b_n$, THEN $\forall m \in \mathbb{N}, \quad a_m < b_m$.

2. IF $a_n << b_n$, THEN $\exists m \in \mathbb{N}, \quad a_m < b_m$.

3. IF $a_n << b_n$, THEN $\exists n_0 \in \mathbb{N}$,
   $\forall m \in \mathbb{N}, \quad m \geq n_0 \implies a_m < b_m$.

4. IF $a_n << b_n$, THEN $\forall \varepsilon > 0, \ \exists n_0 \in \mathbb{N}$,
   $\forall m \in \mathbb{N}, \quad m \geq n_0 \implies a_m < \varepsilon b_m$.

True or False – The Big Theorem

Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be positive sequences.

5. IF $\forall m \in \mathbb{N},\ a_m < b_m,$ THEN $a_n << b_n$.

6. IF $\exists m \in \mathbb{N},\ a_m < b_m,$ THEN $a_n << b_n$.

7. IF $\exists n_0 \in \mathbb{N}, \quad \forall m \in \mathbb{N}, \quad m \geq n_0 \implies a_m < b_m,$
   THEN $a_n << b_n$.

8. IF $\forall \varepsilon > 0, \exists n_0 \in \mathbb{N}, \forall m \in \mathbb{N}, \quad m \geq n_0 \implies a_m < \varepsilon b_m,$
   THEN $a_n << b_n$.

Refining the Big Theorem - 1

1. Construct a sequence $\{u_n\}_n$ such that

$$\begin{cases} \forall a \leq 0, & n^a << u_n \\ \forall a > 0, & u_n << n^a \end{cases}$$

2. Construct a sequence $\{v_n\}_n$ such that

$$\begin{cases} \forall a < 0, & n^a << v_n \\ \forall a \geq 0, & v_n << n^a \end{cases}$$

Refining the Big Theorem - 2

1. Construct a sequence $\{u_n\}_n$ such that

$$\begin{cases} \forall a \leq 2, & n^a << u_n \\ \forall a > 2, & u_n << n^a \end{cases}$$

2. Construct a sequence $\{v_n\}_n$ such that

$$\begin{cases} \forall a < 2, & n^a << v_n \\ \forall a \geq 2, & v_n << n^a \end{cases}$$