MAT137 - Calculus with proofs

- Assignment 7 due tomorrow
- Assignment 8 due on March 4
- Test 4 opens on March 12

- Today: Theorems about sequences
- Friday: The Big Theorem
  (**Watch Videos 11.7, 11.8**)

True or False - Rapid fire

1. (convergent) $\implies$ (bounded)
2. (convergent) $\implies$ (monotonic)
3. (convergent) $\implies$ (eventually monotonic)
4. (bounded) $\implies$ (convergent)
5. (monotonic) $\implies$ (convergent)
6. (bounded + monotonic) $\implies$ (convergent)
7. (divergent to ∞) $\implies$ (eventually monotonic)
8. (divergent to ∞) $\implies$ (unbounded above)
9. (unbounded above) $\implies$ (divergent to ∞)

Proof of Theorem 3

Write a proof for the following Theorem

Theorem 3

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence.

- IF $\{a_n\}_{n=0}^{\infty}$ is increasing AND unbounded above,
- THEN $\{a_n\}_{n=0}^{\infty}$ is divergent to ∞

1. Write the definitions of "increasing", "unbounded above", and "divergent to ∞"
2. Using the definition of what you want to prove, write down the structure of the formal proof.
3. Do some rough work if necessary.
4. Write a formal proof.

Proof feedback

1. Does your proof have the correct structure?
2. Are all your variables fixed (not quantified)? In the right order? Do you know what depends on what?
3. Is the proof self-contained? Or do I need to read the rough work to understand it?
4. Does each statement follow logically from previous statements?
5. Did you explain what you were doing? Would your reader be able to follow your thought process without reading your mind?

Critique this proof - #1

- $\forall M \in \mathbb{R},\ \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N}, \quad n \geq n_0 \implies x_n > M$

- M is not an upper bound: $\exists n_0 \in \mathbb{N}$ s.t. $x_{n_0} > M$

- $n \geq n_0 \implies x_n \geq x_{n_0} > M$

Critique this proof - #2

- WTS $a_n \to \infty$. This means:
  $\forall M \in \mathbb{R},\ \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N}, \quad n \geq n_0 \implies x_n > M$

- bounded above: $\exists M \in \mathbb{R},\ \forall n \in \mathbb{N},\ x_n \leq M$

- negation: $\forall M \in \mathbb{R},\ \exists n \in \mathbb{N},\ x_n > M$

- $\forall n \in \mathbb{N}$, take $n = n_0$.