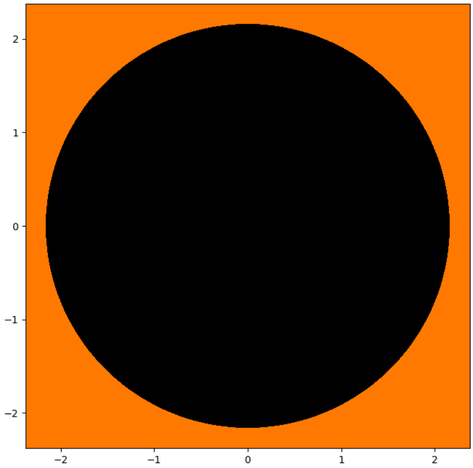
Filled Julia sets of complex quadratic maps



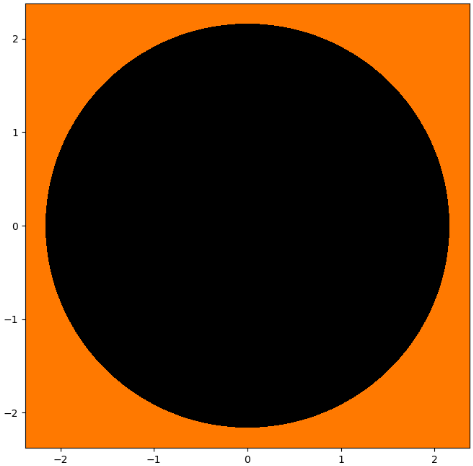
$c = -2.5$

- Points further than

$$\frac{1}{2} + \sqrt{\frac{1}{4} + d(c, 0)}$$

  from 0 always fly off toward ∞.
- These points form a set L_0.
- The points that reach L_0 after n steps form a set L_n.

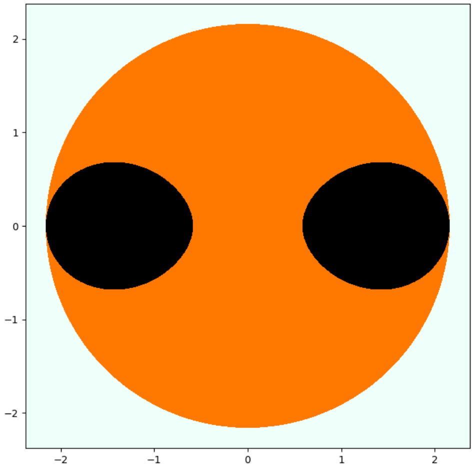
Filled Julia sets of complex quadratic maps



$c = -2.5$

- Cut out L_0.

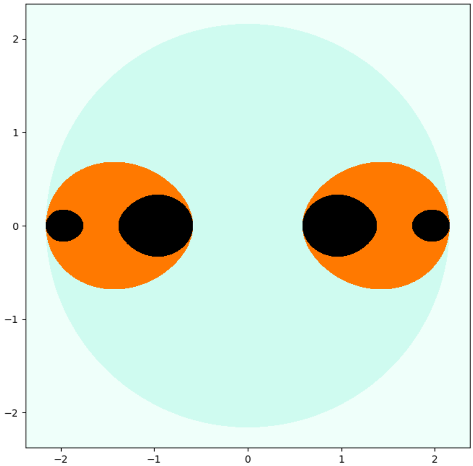
Filled Julia sets of complex quadratic maps



$c = -2.5$

- Cut out L_0.
- Cut out L_1.

Filled Julia sets of complex quadratic maps

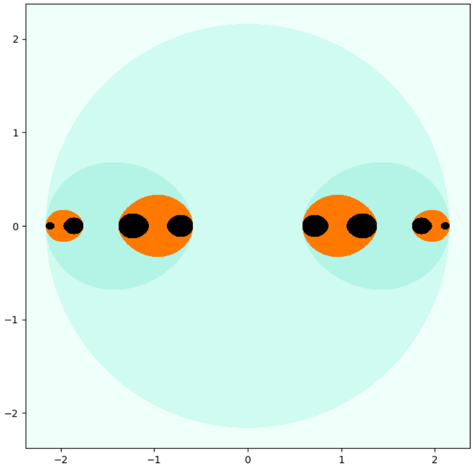


$c = -2.5$

- Cut out L_0.
- Cut out L_1.
- Cut out L_2.

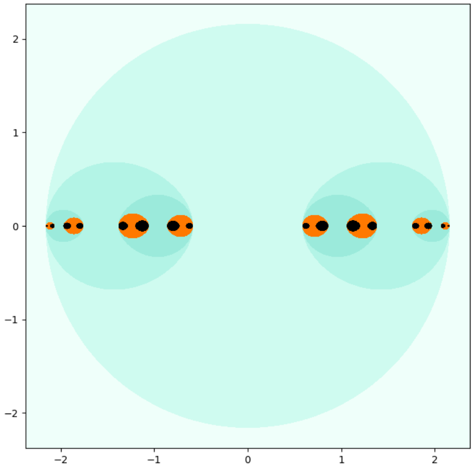
Filled Julia sets of complex quadratic maps

$c = -2.5$



- Cut out L_0.
- Cut out L_1.
- Cut out L_2.
- Cut out L_3.

Filled Julia sets of complex quadratic maps

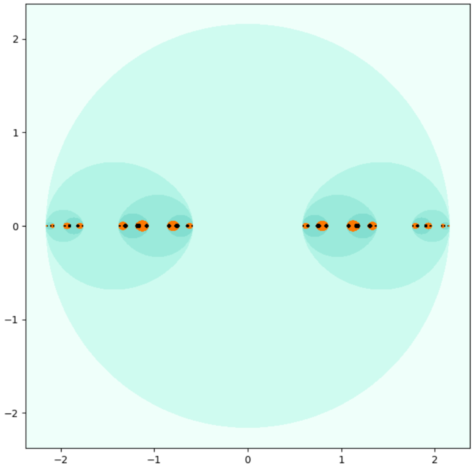


$c = -2.5$

- Cut out L_0.
- Cut out L_1.
- Cut out L_2.
- Cut out L_3.
- Cut out L_4.

Filled Julia sets of complex quadratic maps

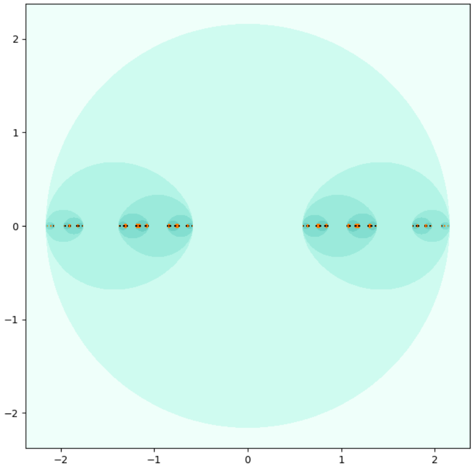
$c = -2.5$



- Cut out L_0.
- Cut out L_1.
- Cut out L_2.
- Cut out L_3.
- Cut out L_4.
- Cut out L_5.

Filled Julia sets of complex quadratic maps

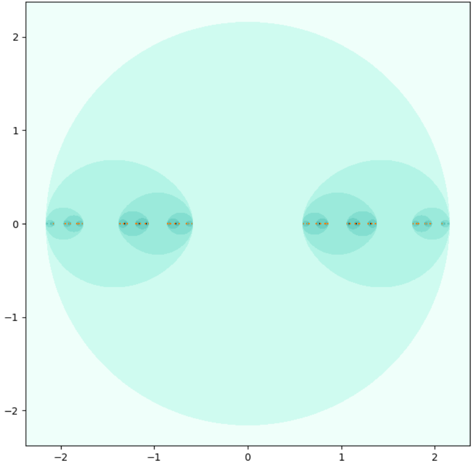
$c = -2.5$



- Cut out L_0.
- Cut out L_1.
- Cut out L_2.
- Cut out L_3.
- Cut out L_4.
- Cut out L_5.
- Cut out L_6.

Filled Julia sets of complex quadratic maps

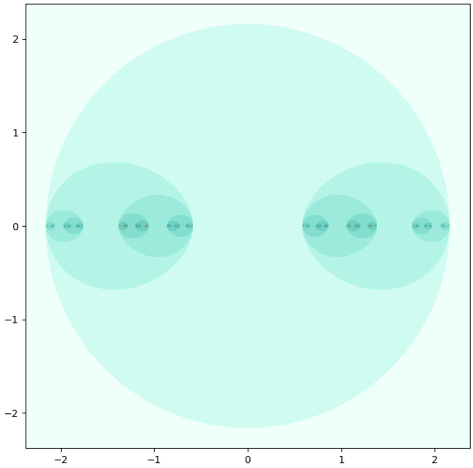
$c = -2.5$



- Cut out L_0.
- Cut out L_1.
- Cut out L_2.
- Cut out L_3.
- Cut out L_4.
- Cut out L_5.
- Cut out L_6.
- Cut out L_7.

Filled Julia sets of complex quadratic maps

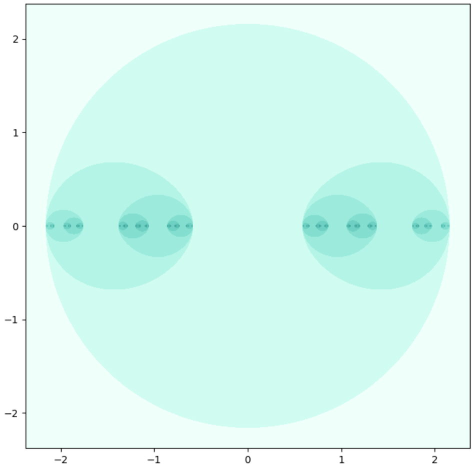
$$c = -2.5$$



- Cut out L_0.
- Cut out L_1.
- Cut out L_2.
- Cut out L_3.
- Cut out L_4.
- Cut out L_5.
- Cut out L_6.
- Cut out L_7.
- $\vdots$

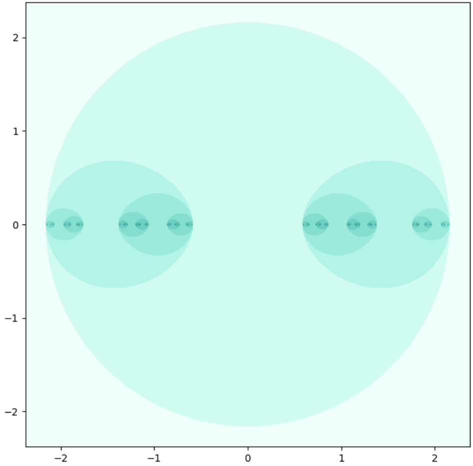
Filled Julia sets of complex quadratic maps

$c = -2.5$



- Cut out L_0.
- Cut out L_1.
- Cut out L_2.
- Cut out L_3.
- Cut out L_4.
- Cut out L_5.
- Cut out L_6.
- Cut out L_7.
- $\vdots$

Filled Julia sets of complex quadratic maps



$$c = -2.5$$

- When c is in the lower region,

$$Q_c : \mathbb{C} \to \mathbb{C}$$

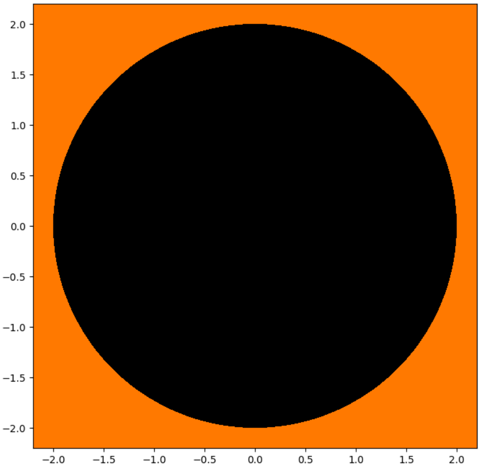
  has the same filled Julia set as

$$Q_c : \mathbb{R} \to \mathbb{R}.$$

- We get a nice way to visualize this weird subset of $\mathbb{R}$.

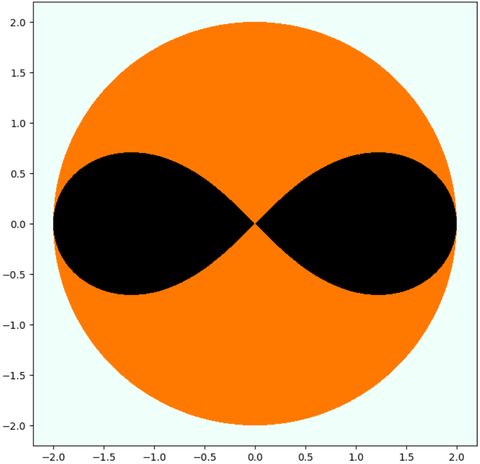
Filled Julia sets of complex quadratic maps

$c = -2$



Filled Julia sets of complex quadratic maps

$c = -2$



Filled Julia sets of complex quadratic maps

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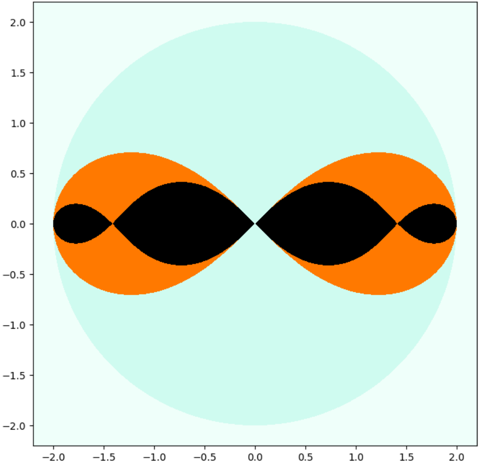
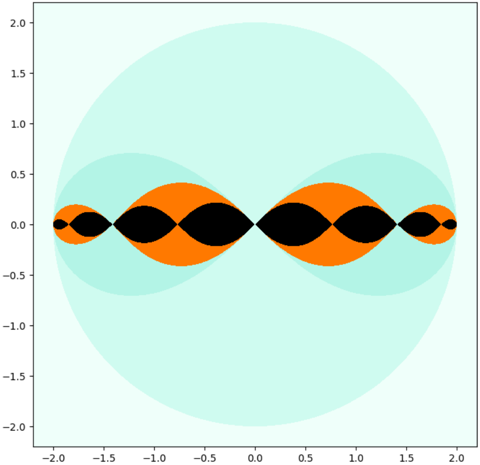


Filled Julia sets of complex quadratic maps

$c = -2$



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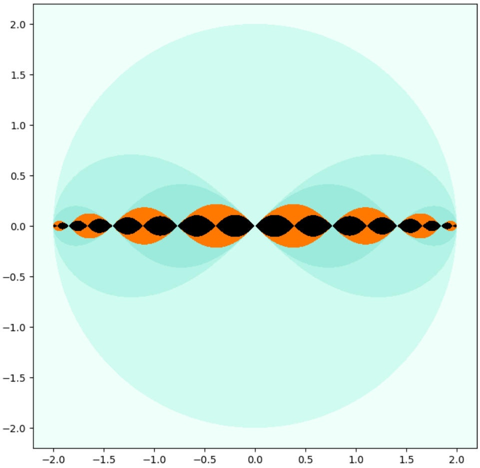

Filled Julia sets of complex quadratic maps

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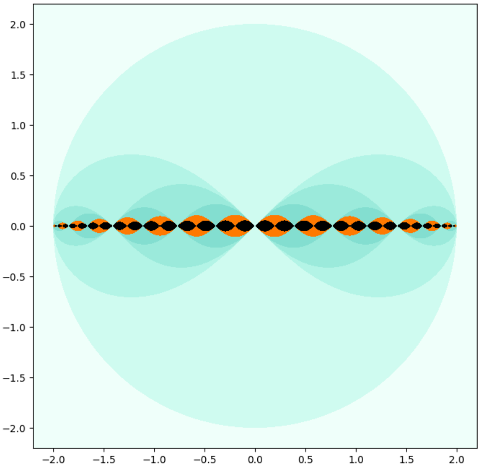

Filled Julia sets of complex quadratic maps

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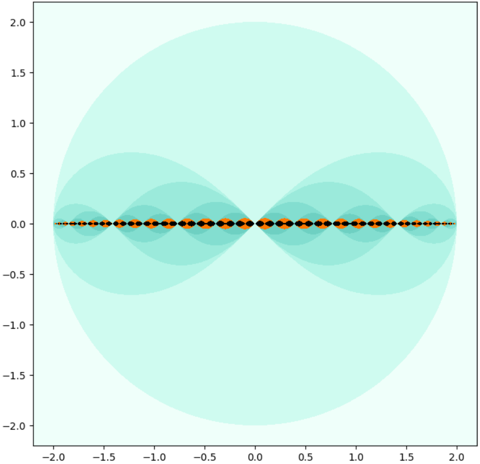

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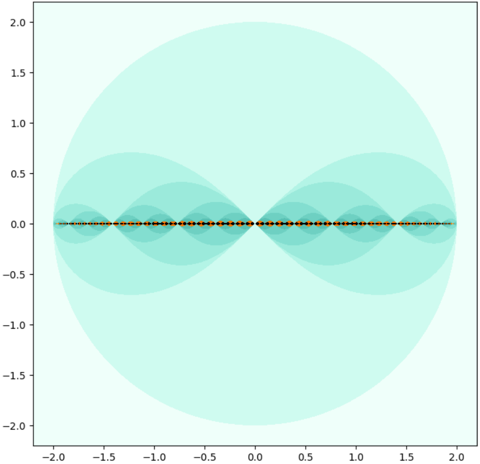

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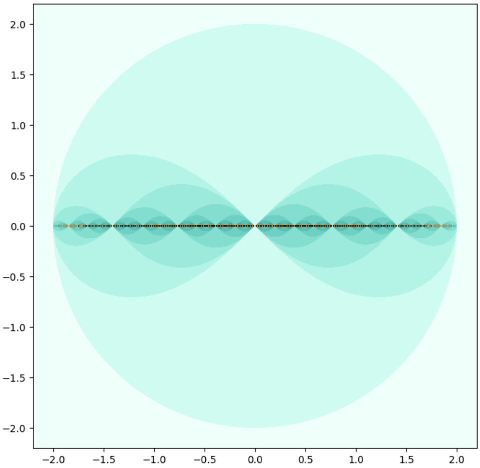
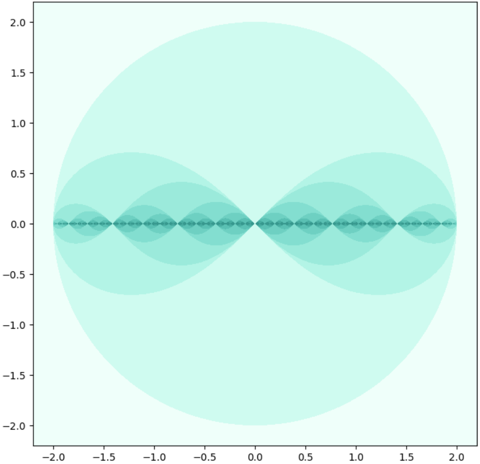


Filled Julia sets of complex quadratic maps



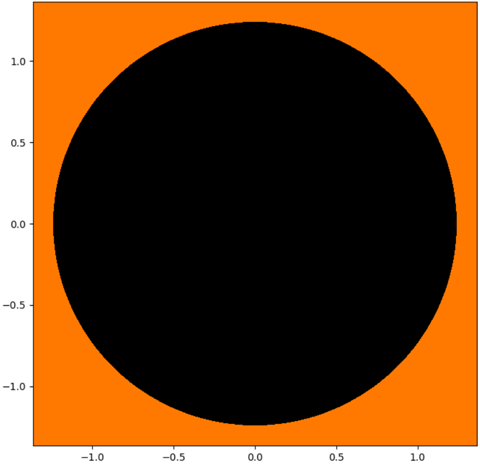
$$c = -2$$

- At the very top of the lower region, the filled Julia set is

$$[-2, 2] \subset \mathbb{R},$$

as expected.

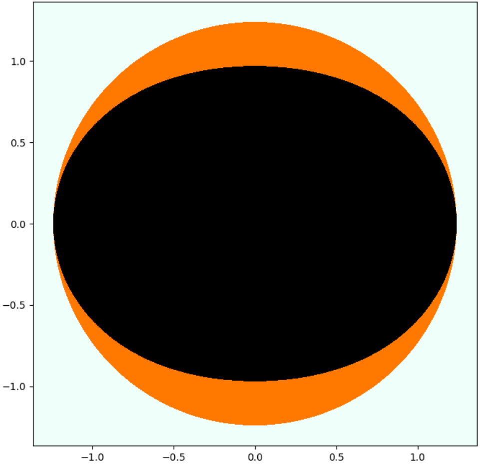
Filled Julia sets of complex quadratic maps



$c = -0.3$

Filled Julia sets of complex quadratic maps

$c = -0.3$



Filled Julia sets of complex quadratic maps

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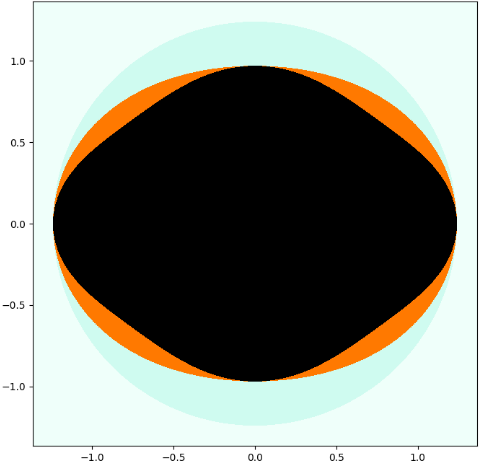

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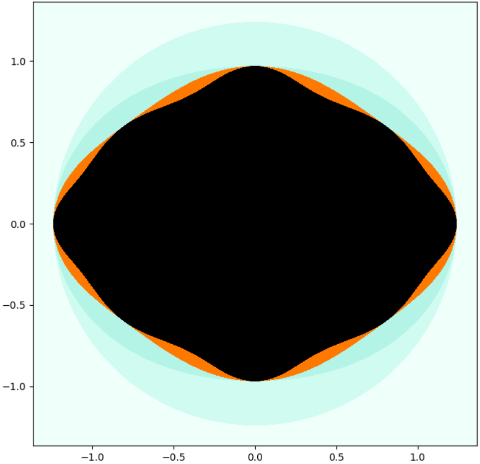

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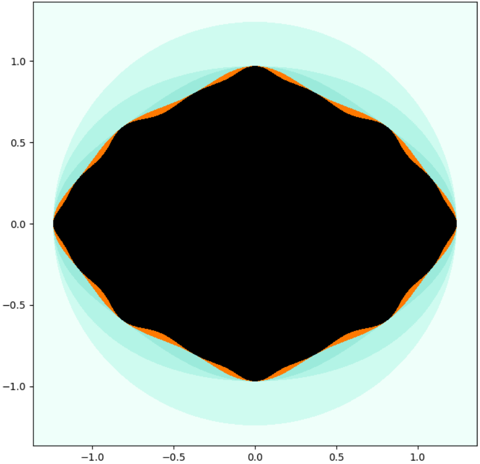

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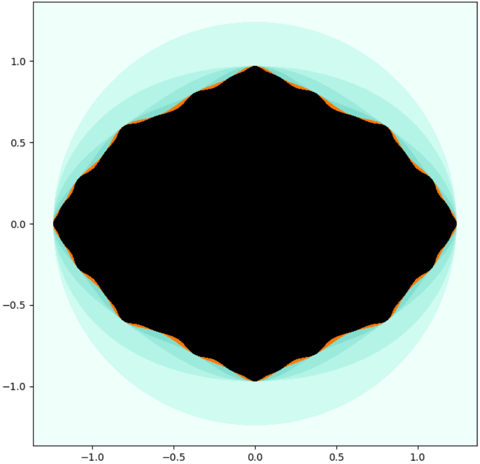

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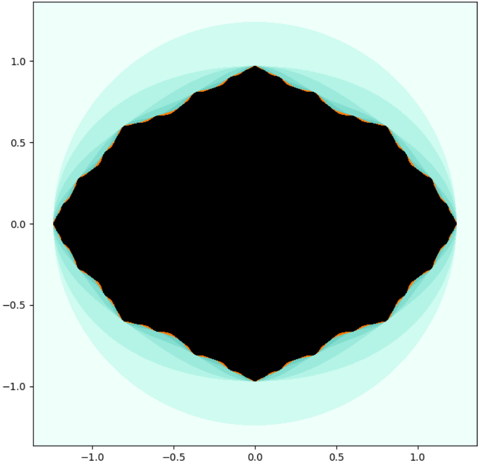

Filled Julia sets of complex quadratic maps

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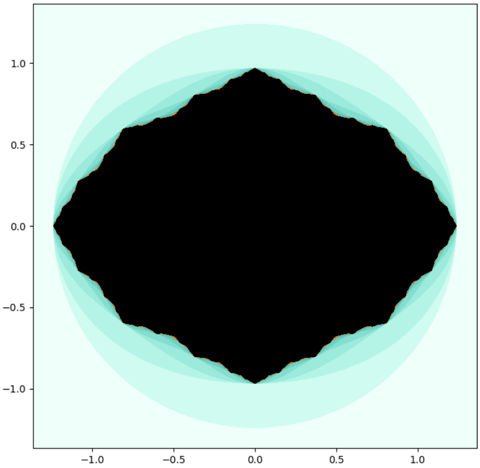

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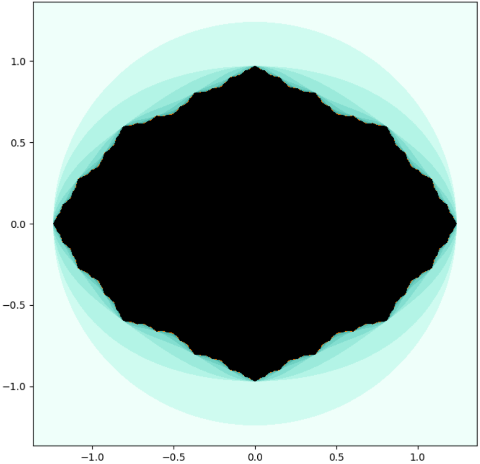
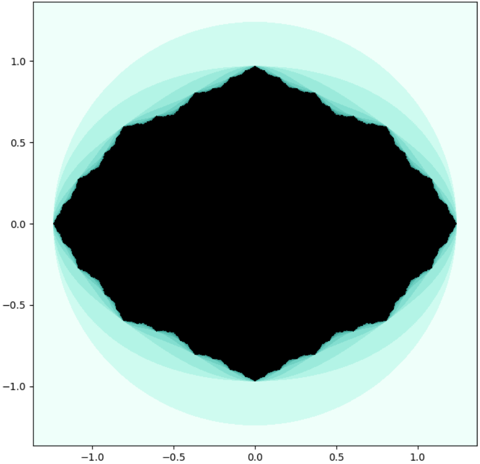


Filled Julia sets of complex quadratic maps

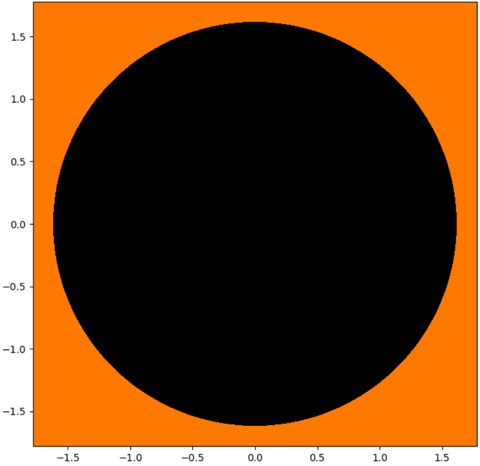


$$c = -0.3$$

- When c is in the upper region, the filled Julia set extends above and below the real axis.

Filled Julia sets of complex quadratic maps

$c = -1$



Filled Julia sets of complex quadratic maps

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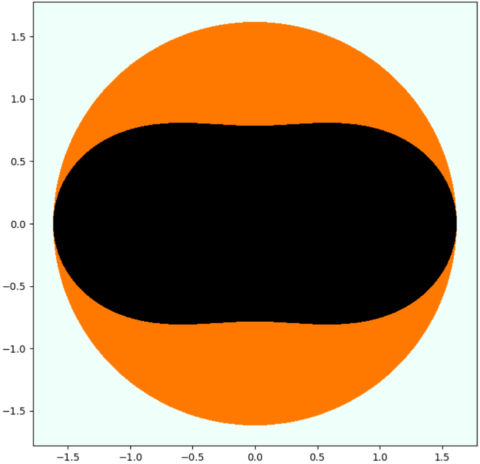

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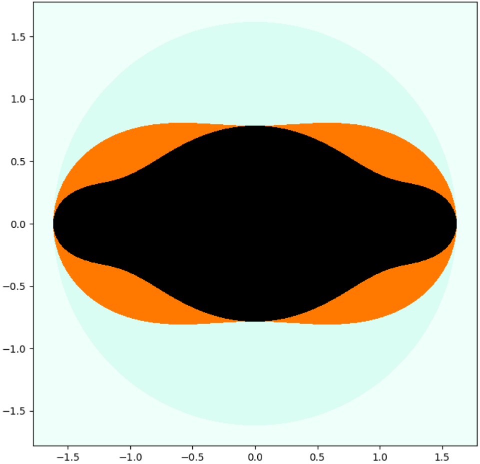

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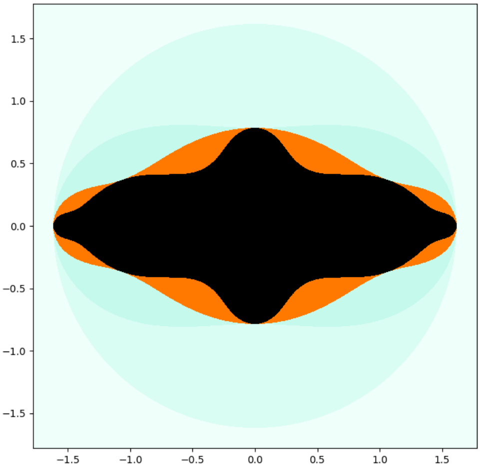

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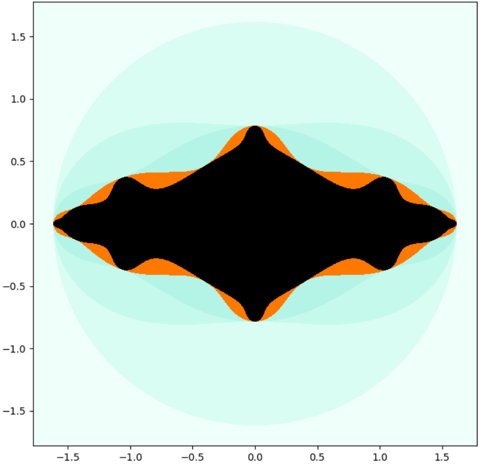

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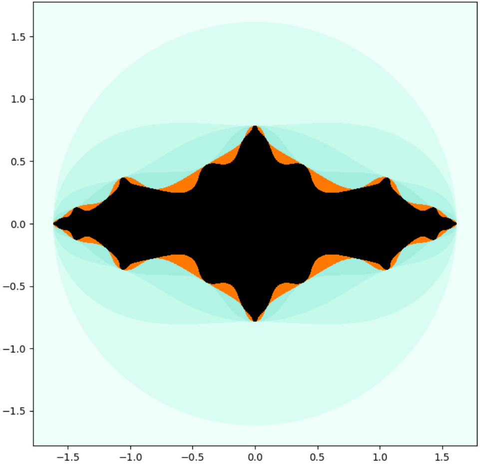

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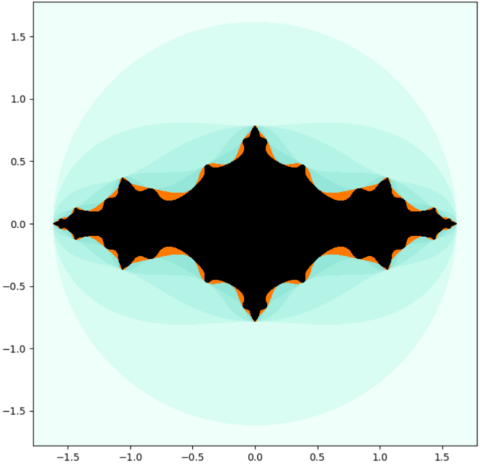

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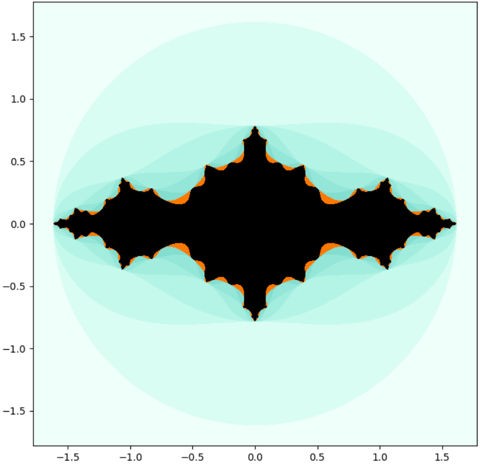

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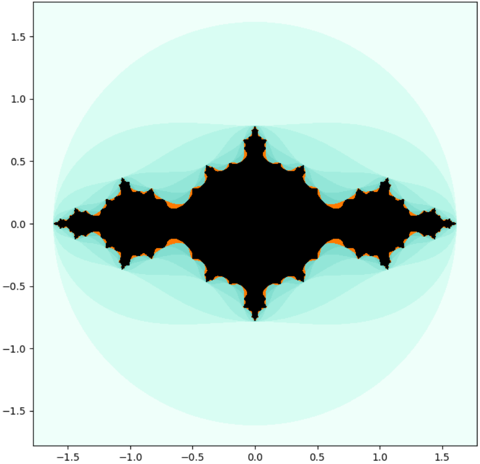

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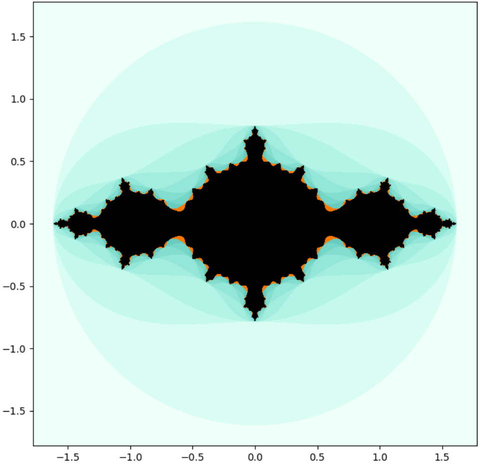

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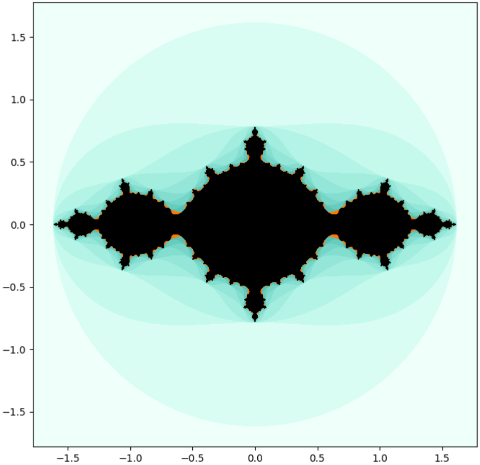

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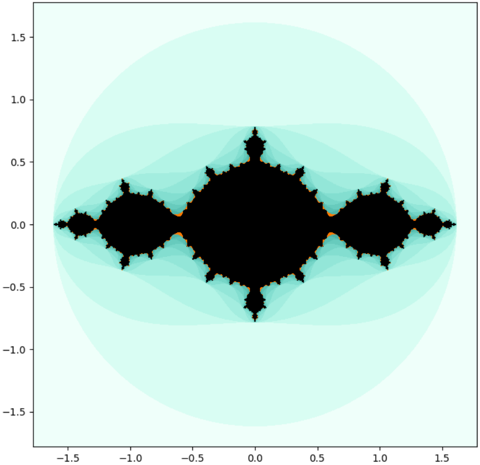

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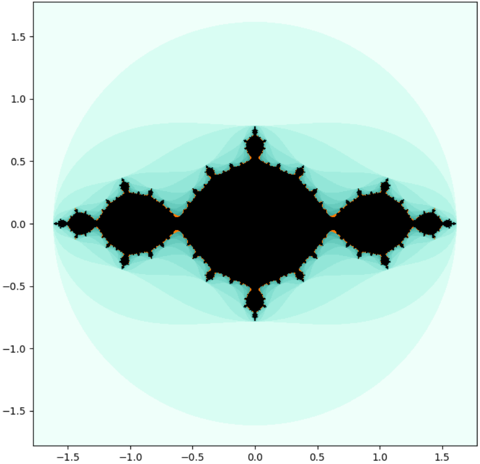

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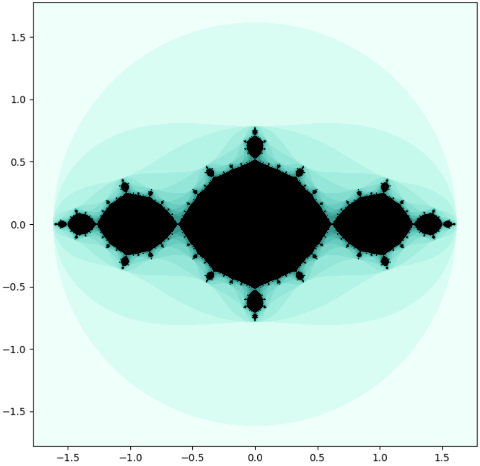
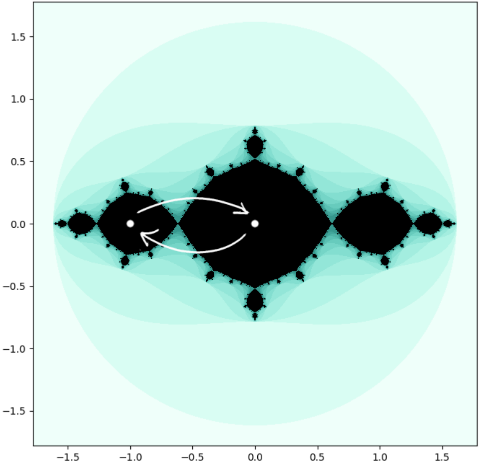


$$c = -1$$

- The people who first studied complex quadratic maps gave fancy names to filled Julia sets they liked.
- This one is called the *basilica*.

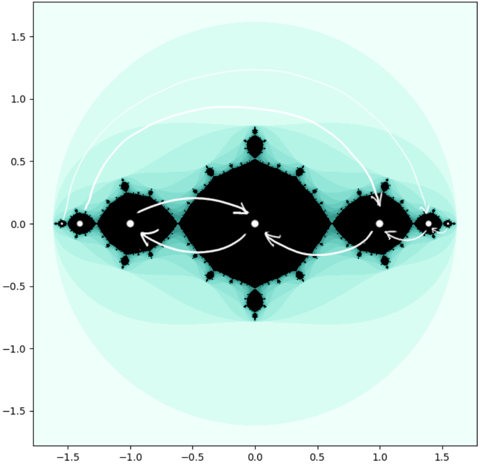
Filled Julia sets of complex quadratic maps



$$c = -1$$

- The people who first studied complex quadratic maps gave fancy names to filled Julia sets they liked.
- This one is called the *basilica*.
- Each lobe contains an eventually periodic point.

Filled Julia sets of complex quadratic maps



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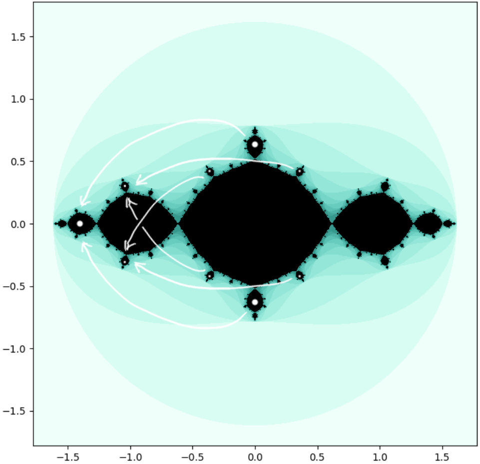

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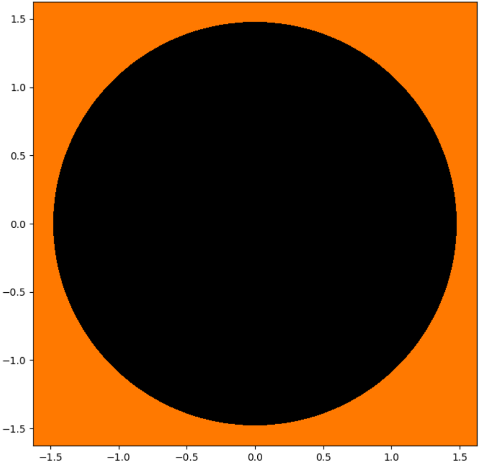
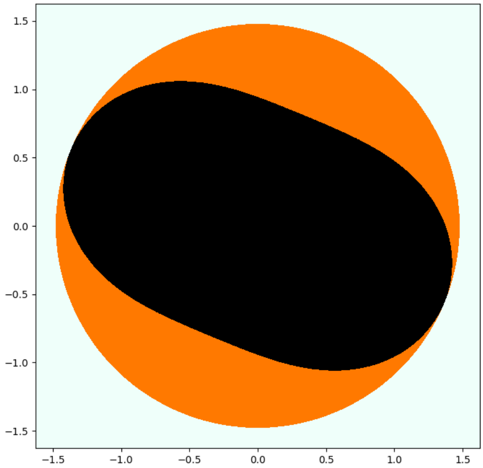


$$c = -0.5 + 0.5i$$

- The constant c doesn't have to be real.

Filled Julia sets of complex quadratic maps



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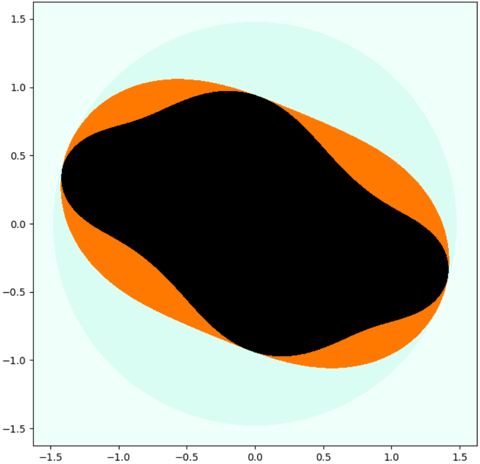

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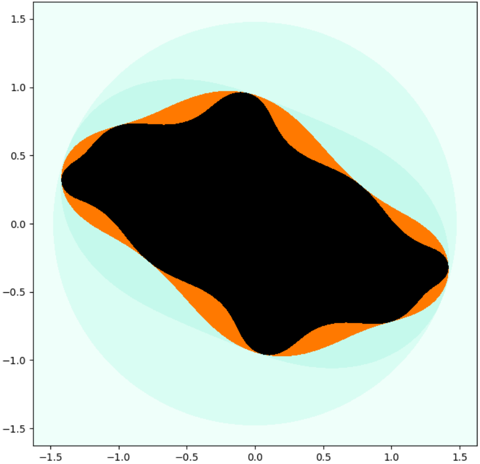

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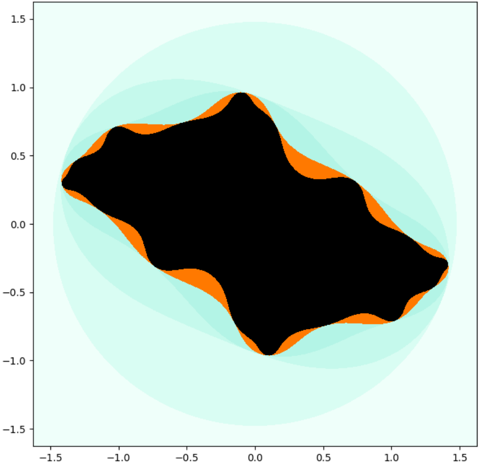

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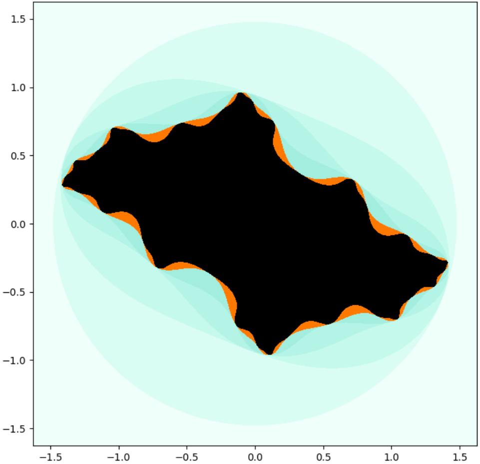

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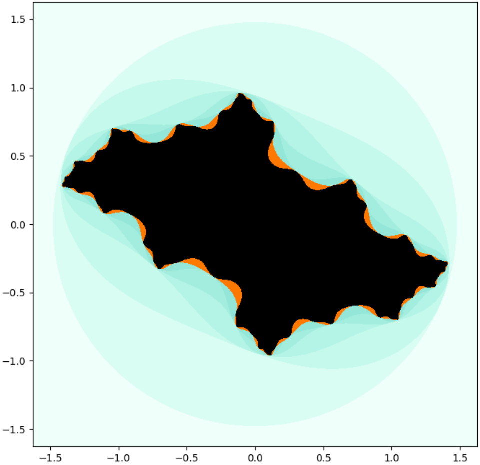

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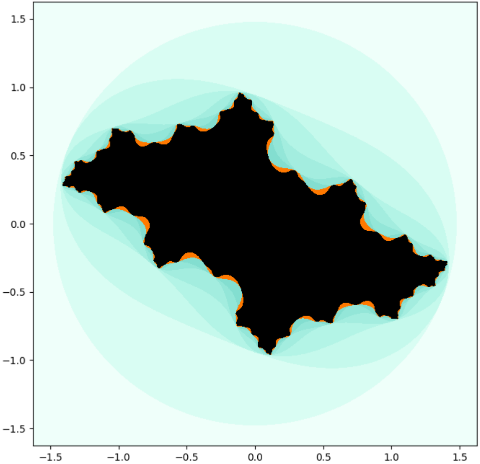
Filled Julia sets of complex quadratic maps



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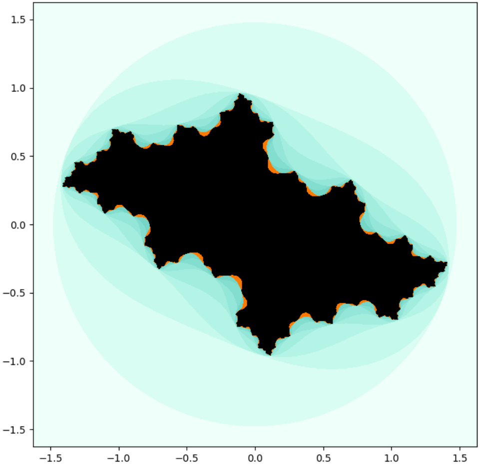
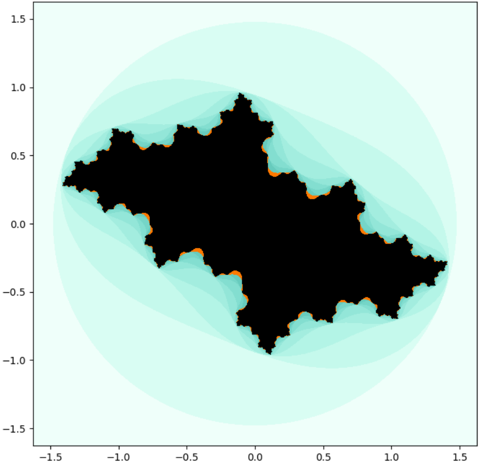
Filled Julia sets of complex quadratic maps



$c = -0.5 + 0.5i$

- The constant c doesn't have to be real.

Filled Julia sets of complex quadratic maps



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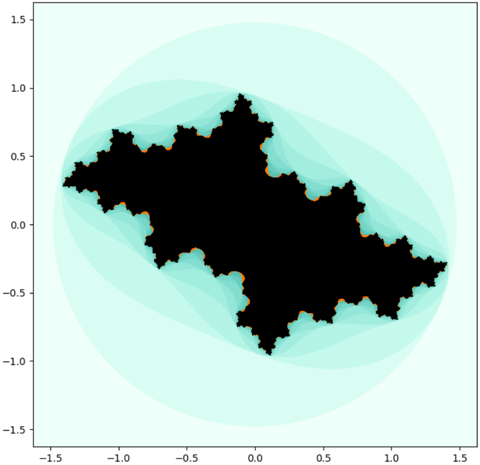

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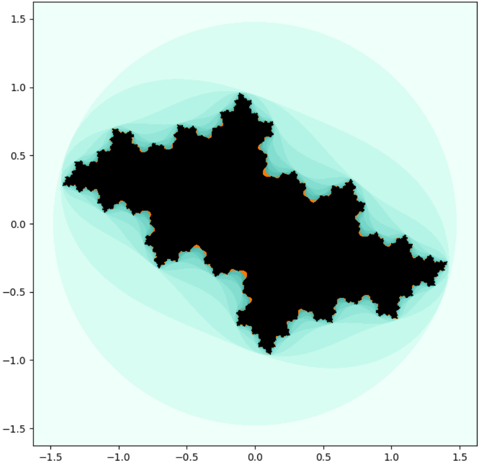

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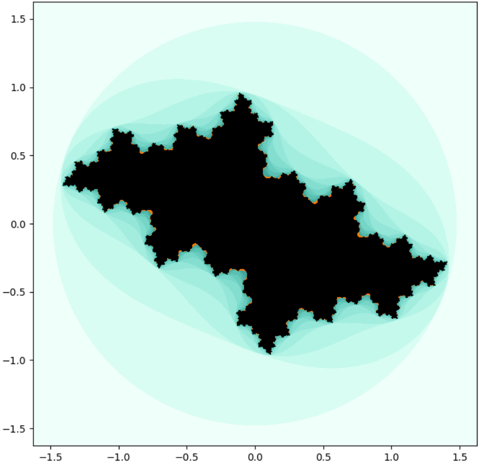

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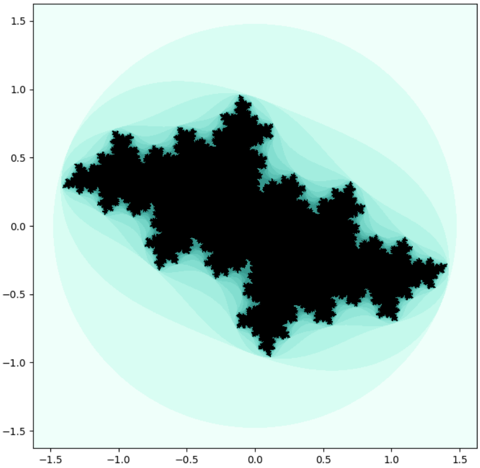

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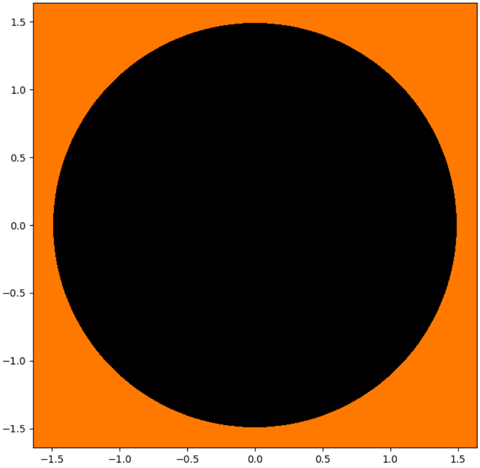

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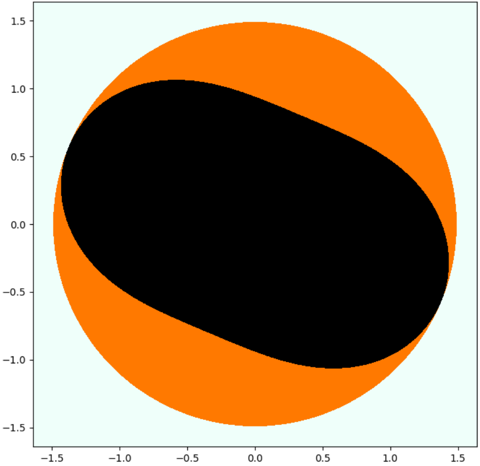

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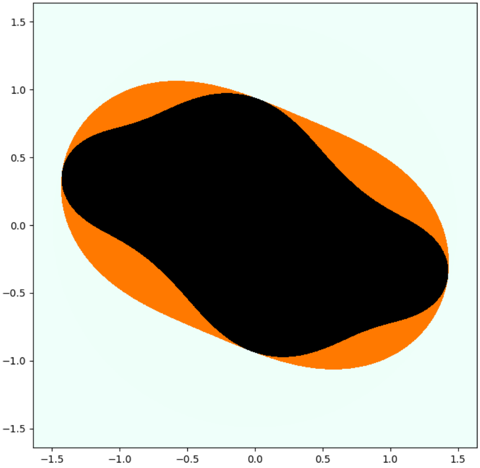

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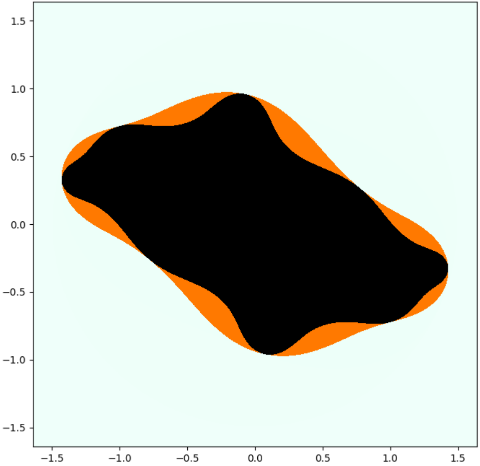

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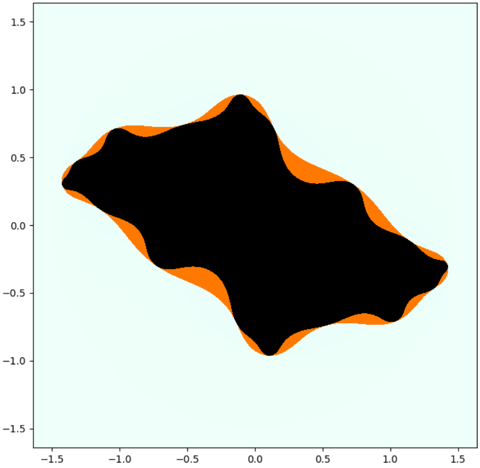

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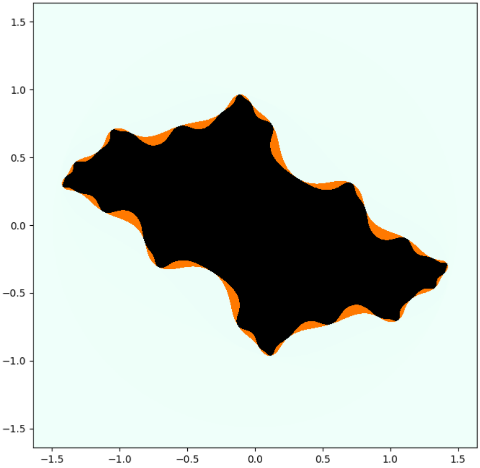

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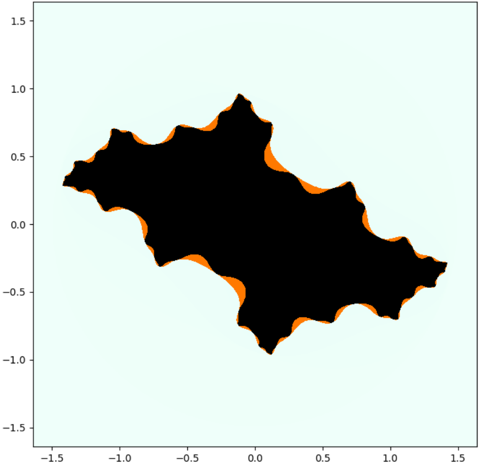

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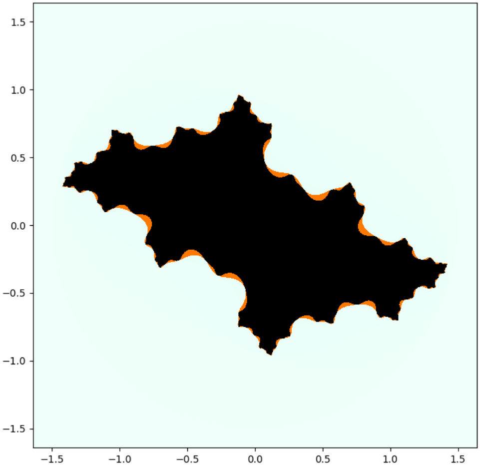

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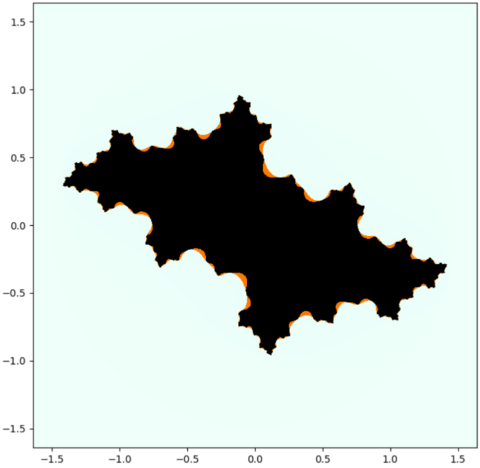

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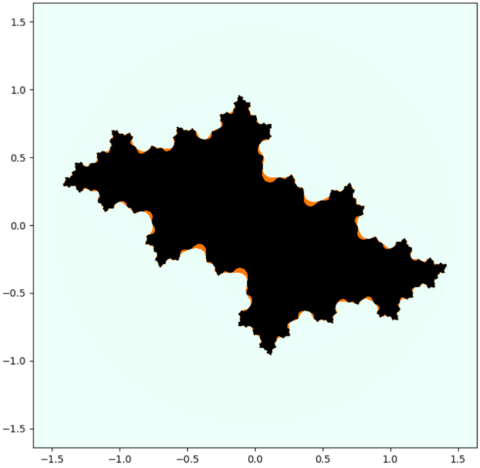

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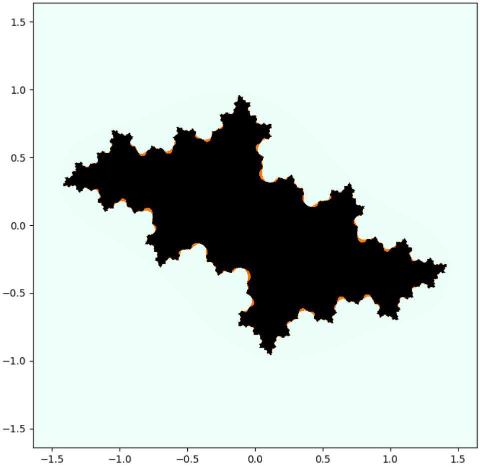

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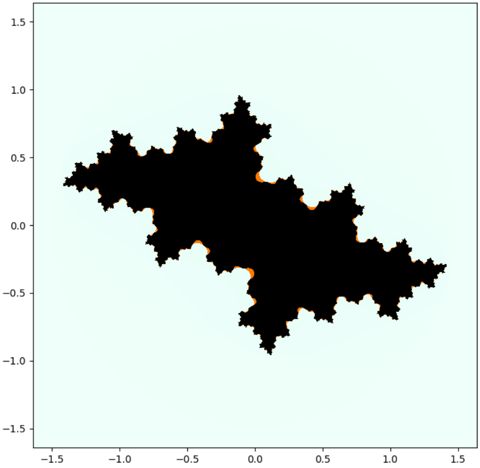

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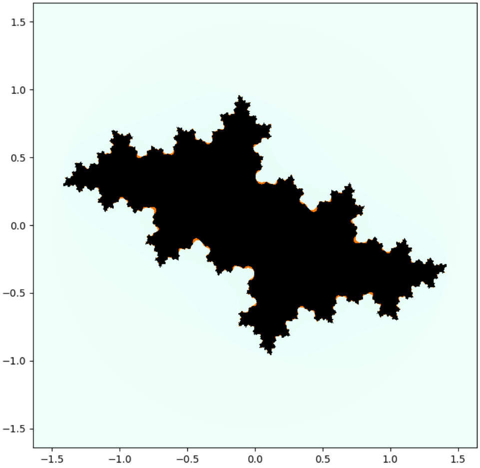

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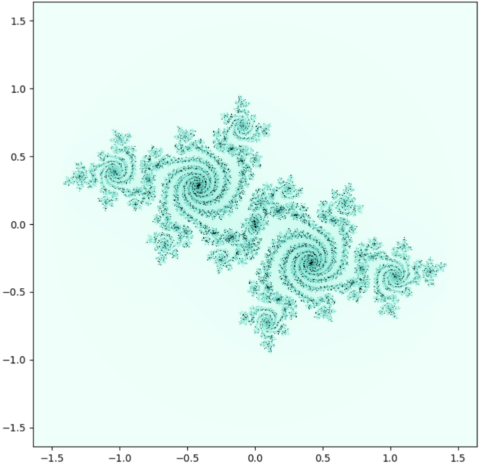

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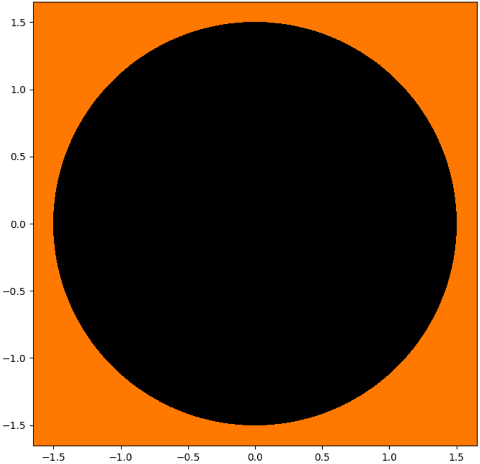
A tour of the zoo



$$c = -0.122 + 0.745i.$$

- Playing with c, you can find all sorts of cool shapes.
- This kind is called a *Douady rabbit*.
- Source: *An Introduction to Chaotic Dynamical Systems*, plates 3–4

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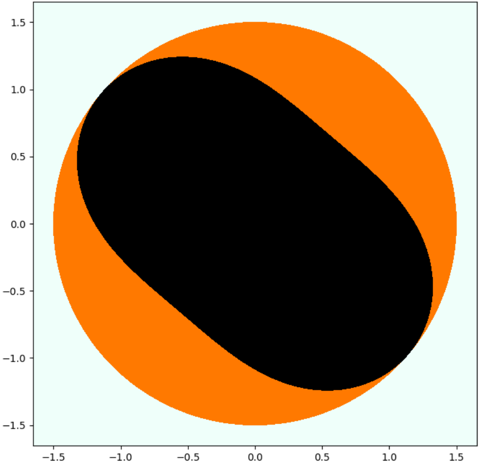

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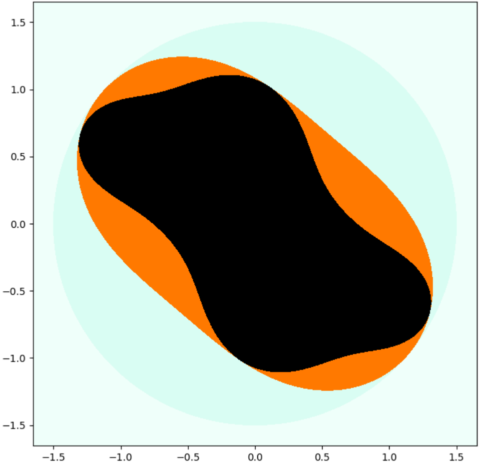

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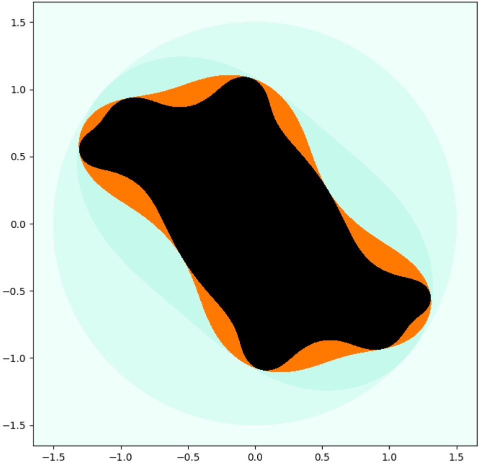

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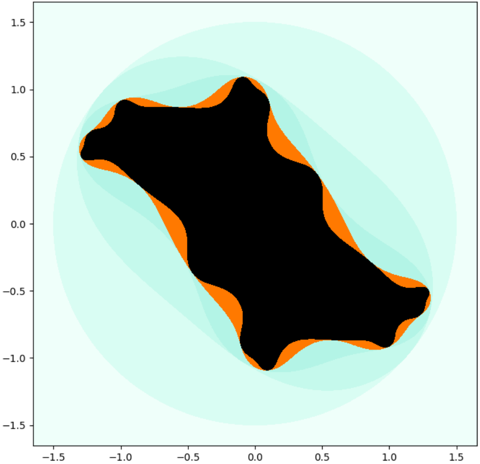

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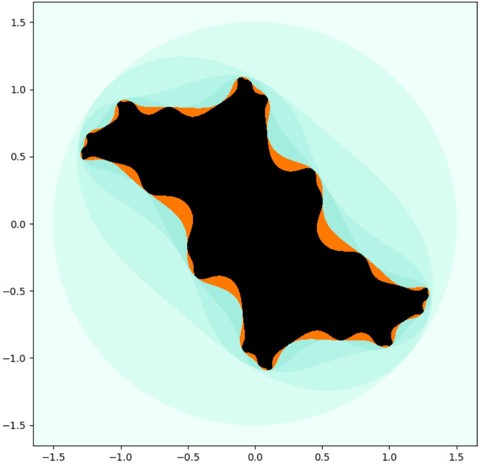

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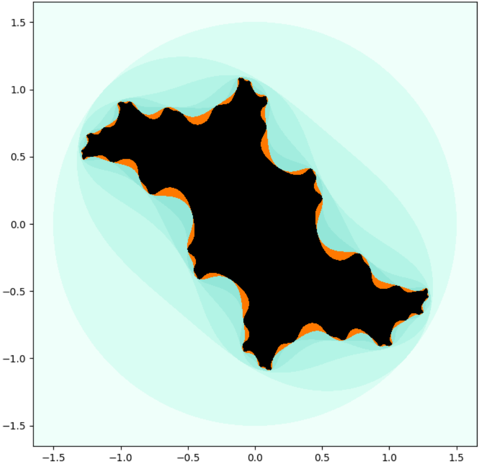

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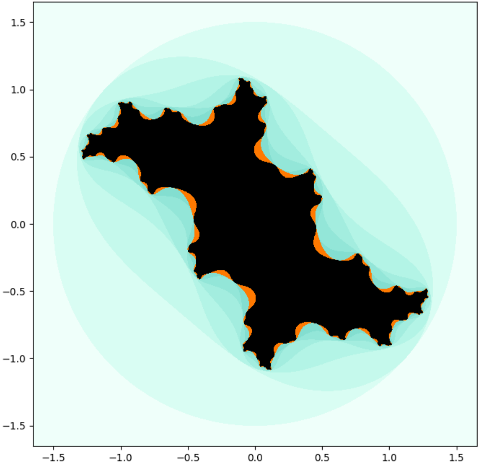

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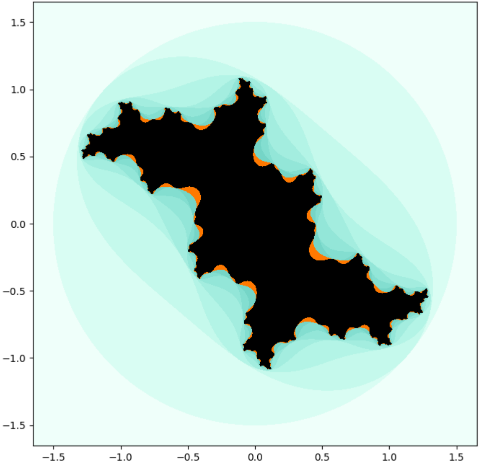

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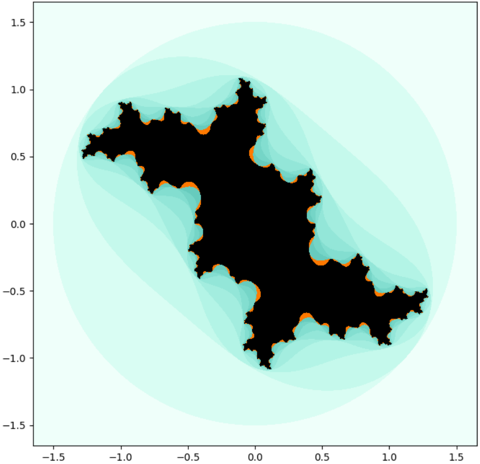

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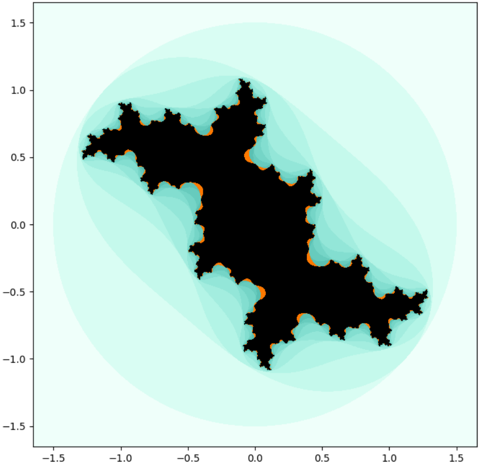

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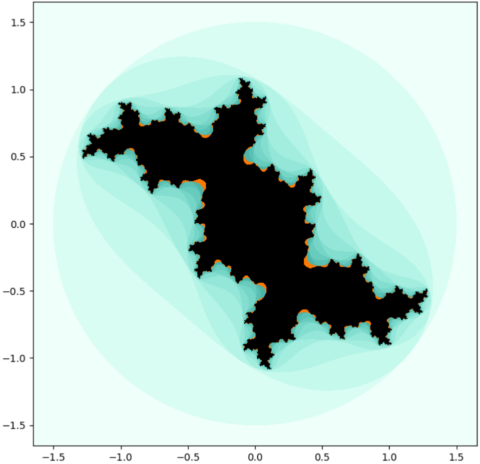

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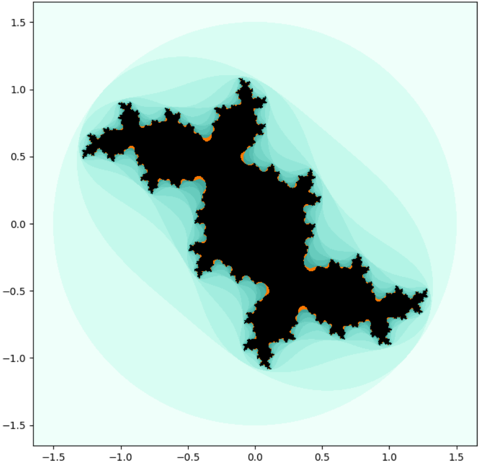

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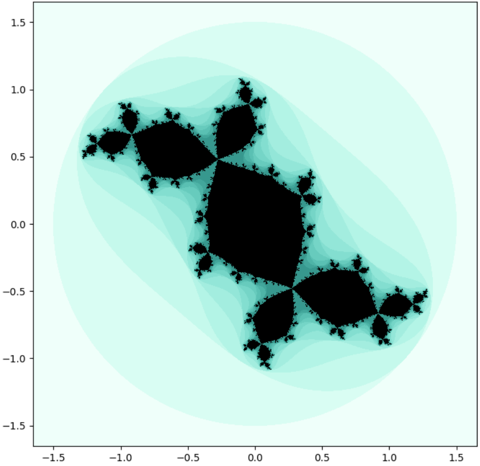

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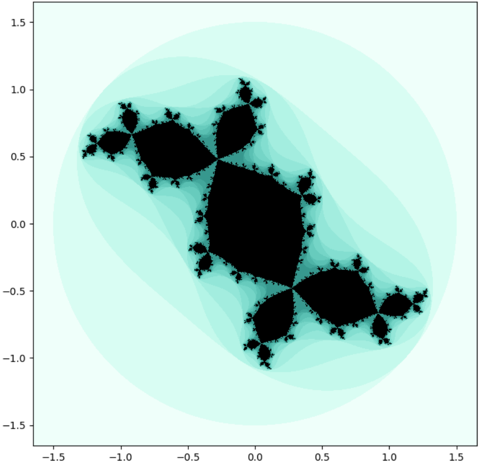

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- It's named after Adrien Douady, one of the first people to study the dynamics of complex quadratic maps.

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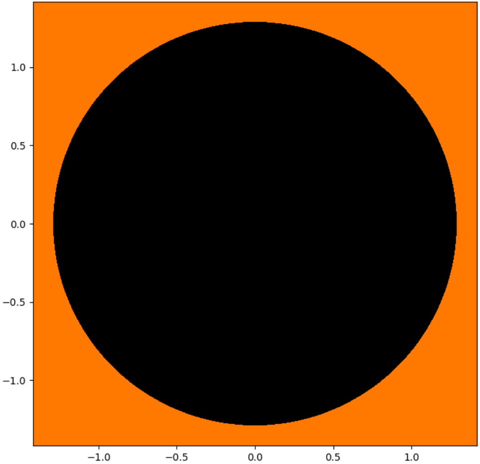

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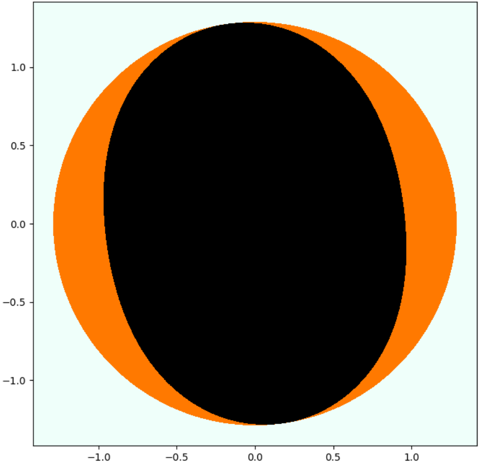

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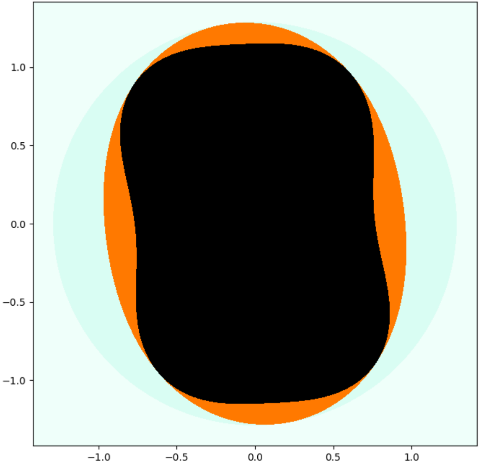

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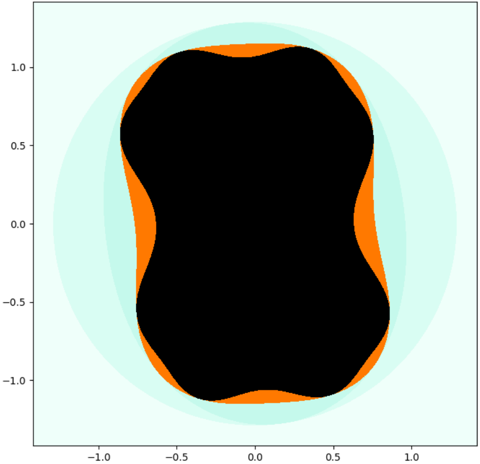

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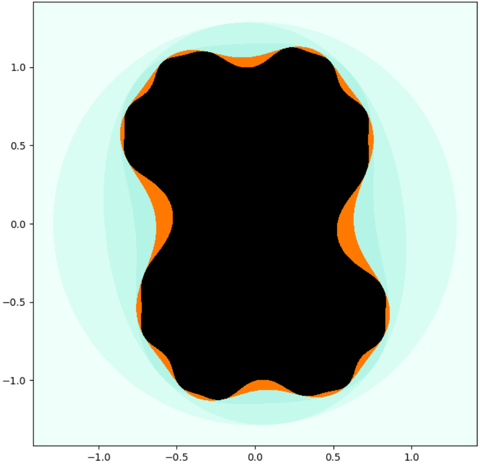

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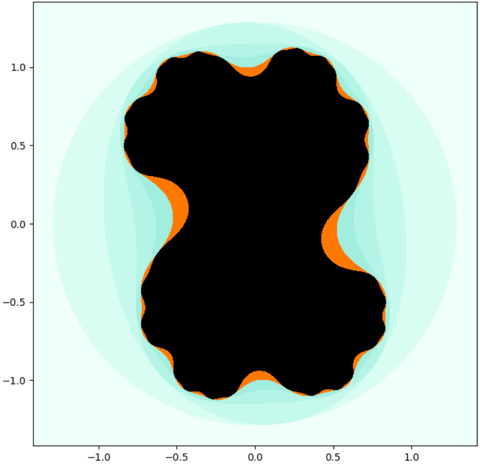

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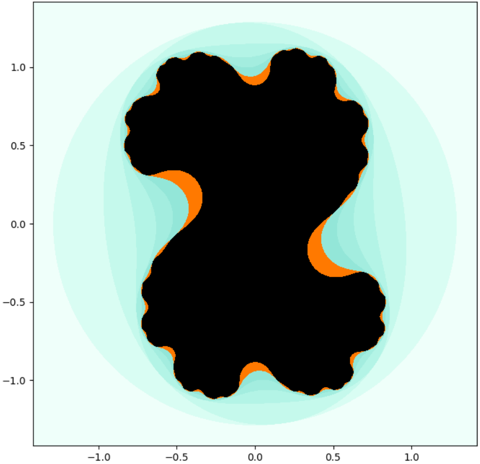

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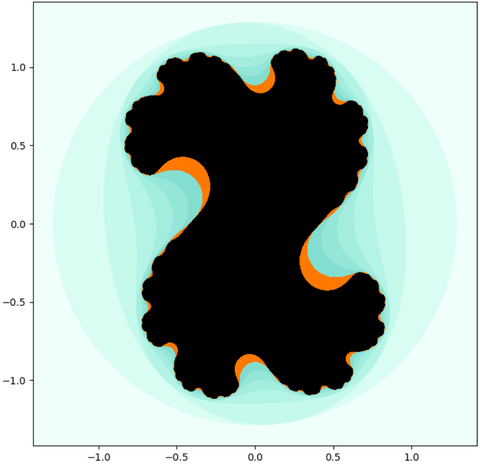

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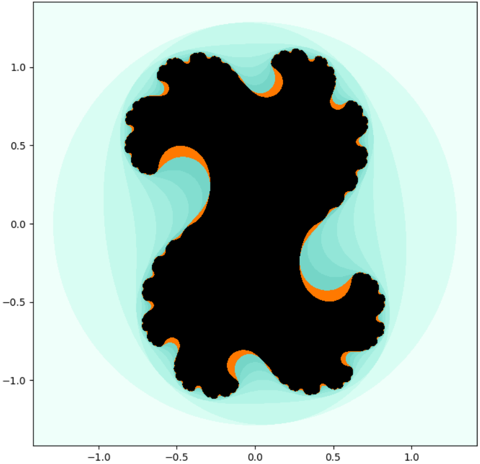

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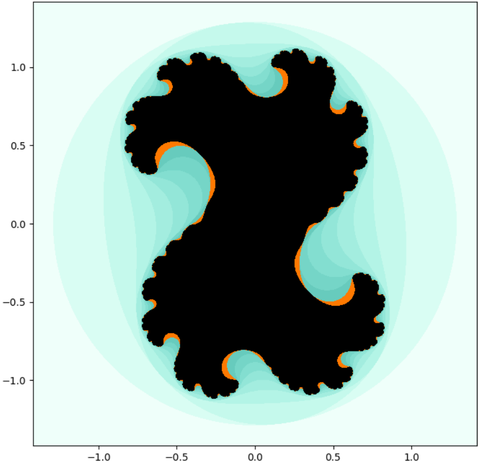

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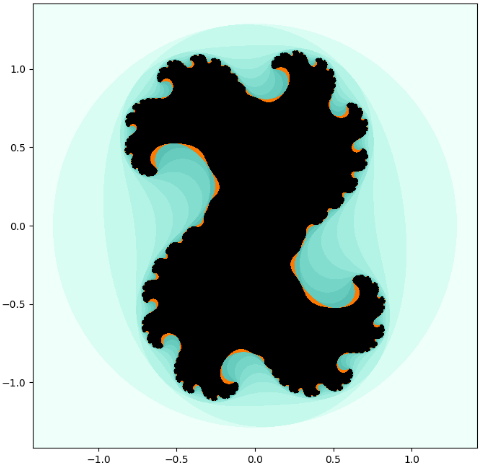

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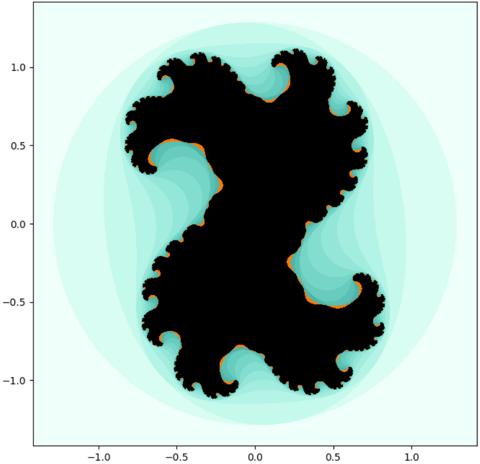

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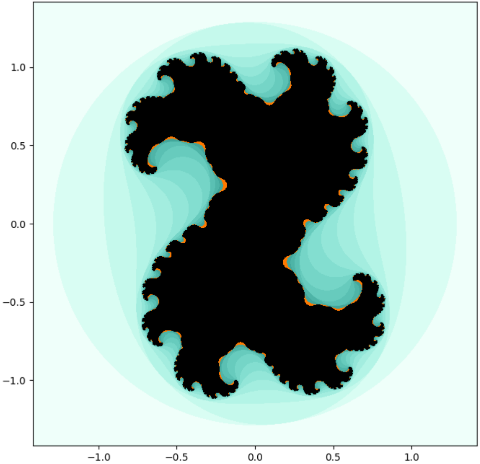
- Source: *An Introduction to Chaotic Dynamical Systems*, plates 1–2

A tour of the zoo



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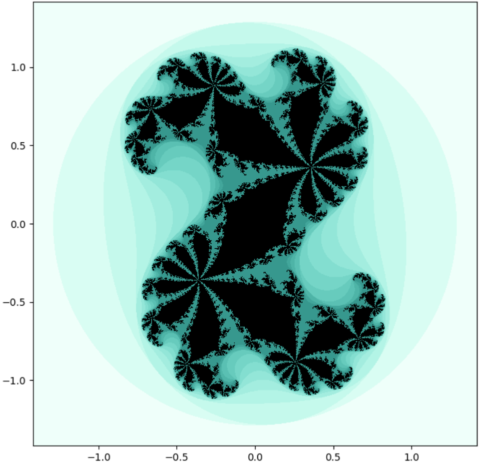

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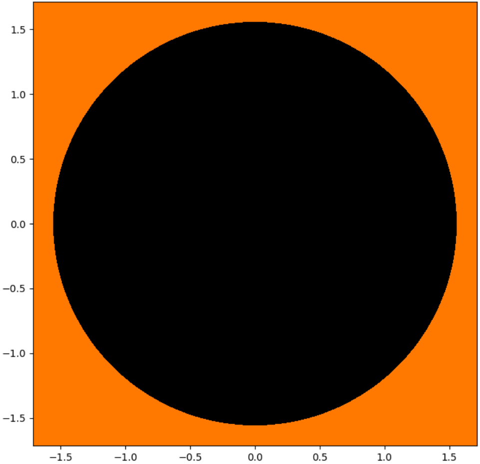

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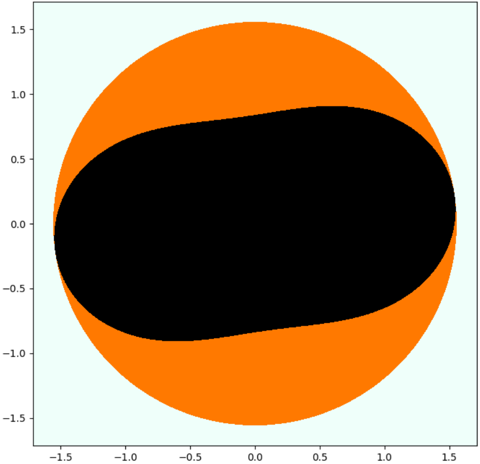

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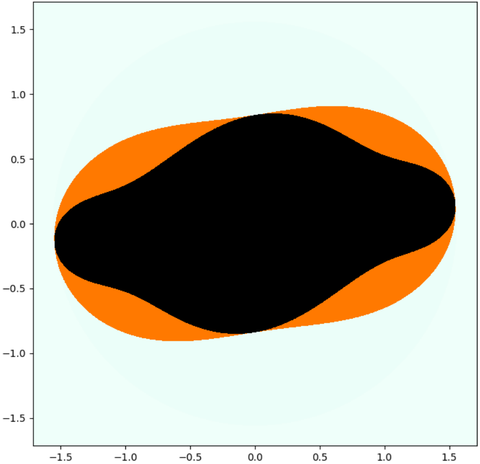

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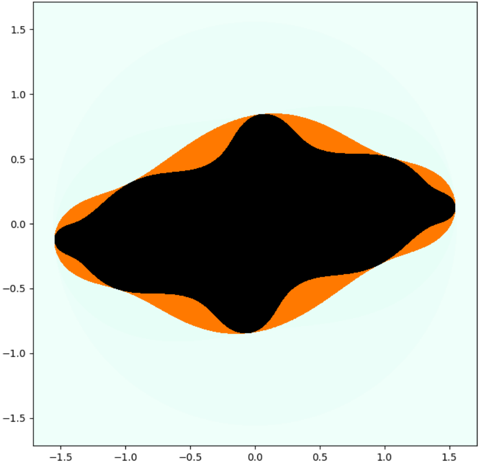

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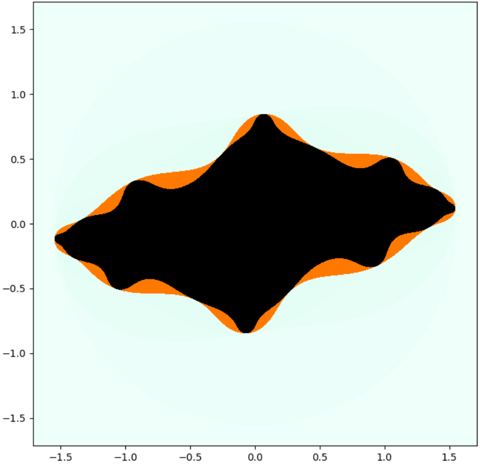

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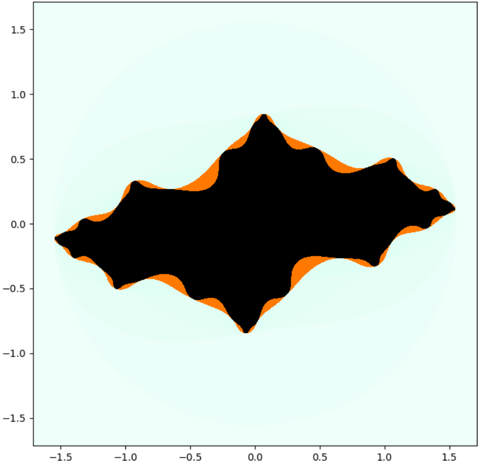

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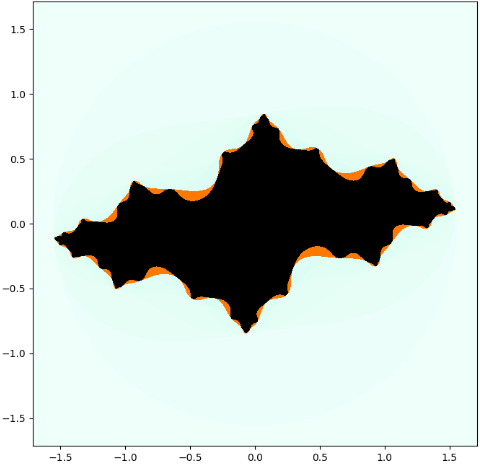

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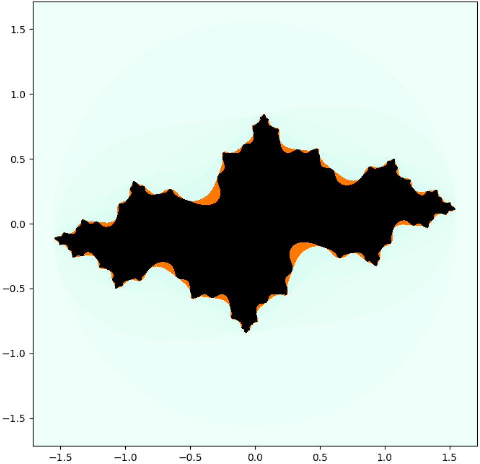

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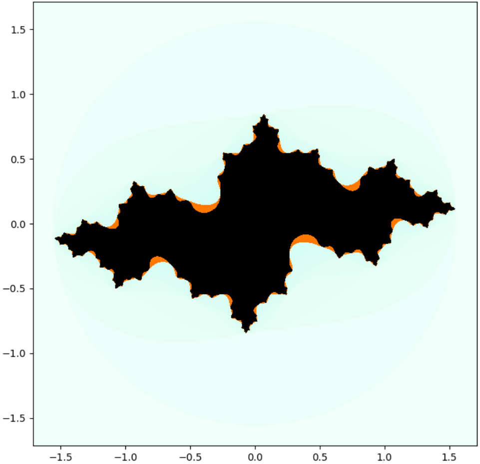

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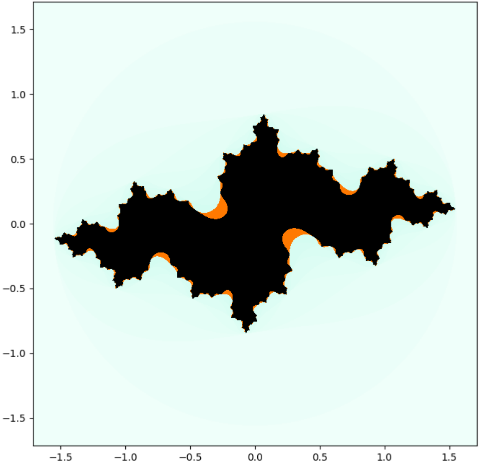

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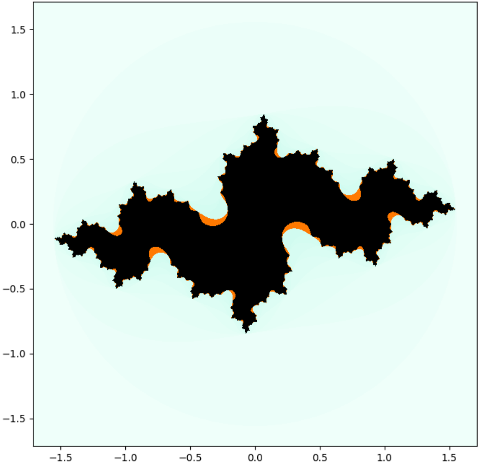

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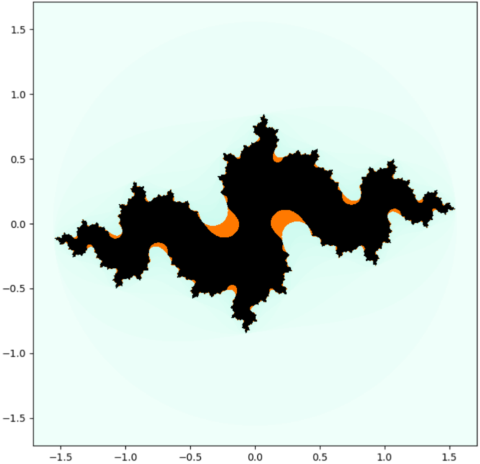

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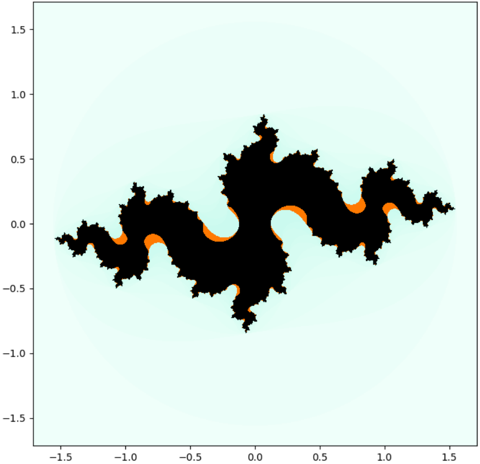

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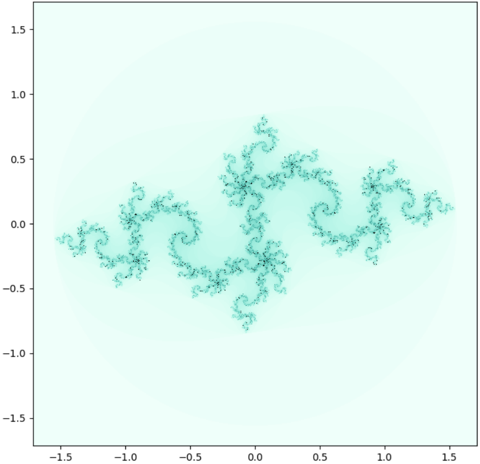
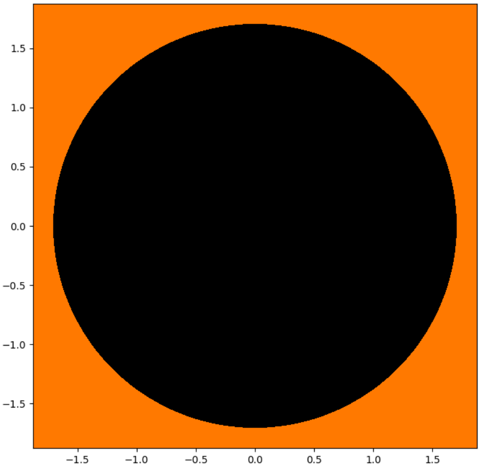


$c = -0.835 - 0.2321i$

- This one starts out looking nice and fat.
- But those early approximations are deceptive!
- Source: Bernardo Galvão de Sousa's MAT 335 notes.

Special kinds of filled Julia sets

$$c = -1 + \frac{2}{3}i$$



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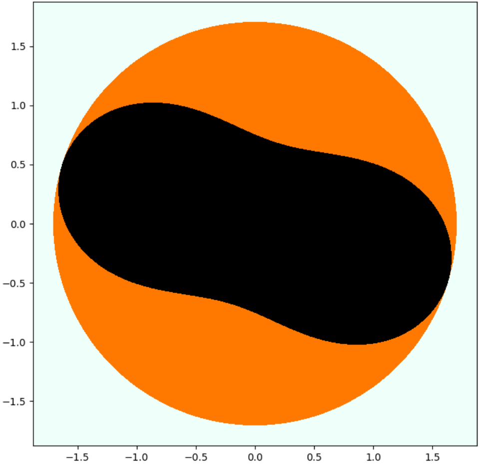

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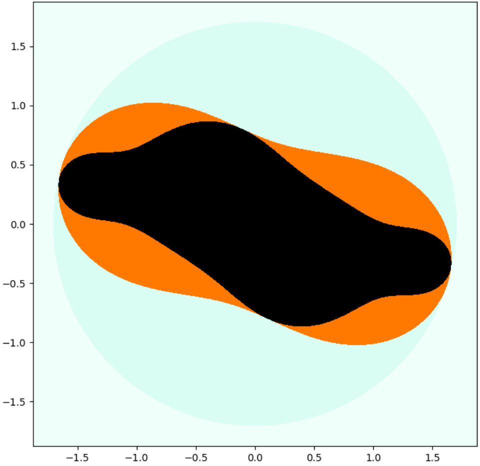

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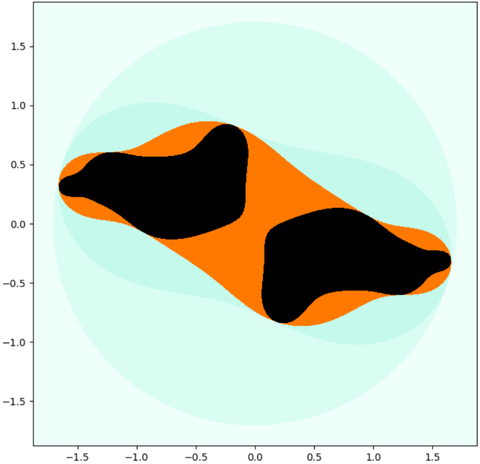
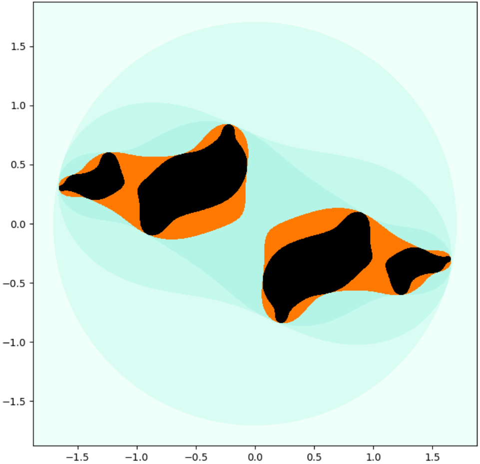


$c = -1 + \frac{2}{3}i$

- Once we cut out $L_0, \ldots, L_3$, we're left with two separate pieces.

Special kinds of filled Julia sets



$$c = -1 + \frac{2}{3}i$$

- Once we cut out $L_0, \ldots, L_3$, we're left with two separate pieces.
- The next cut splits each piece in two.

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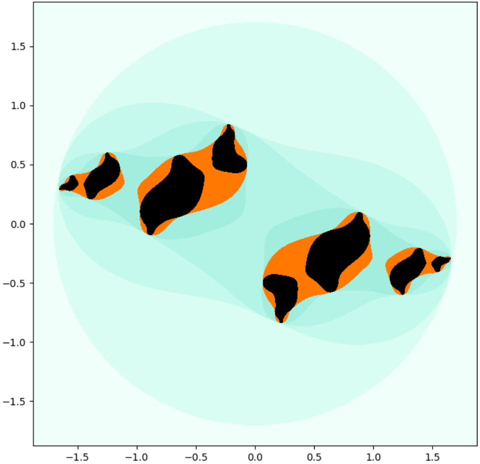

$$c = -1 + \frac{2}{3}i$$

- Once we cut out $L_0, \ldots, L_3$, we're left with two separate pieces.
- The next cut splits each piece in two.
- The next cut does the same thing.

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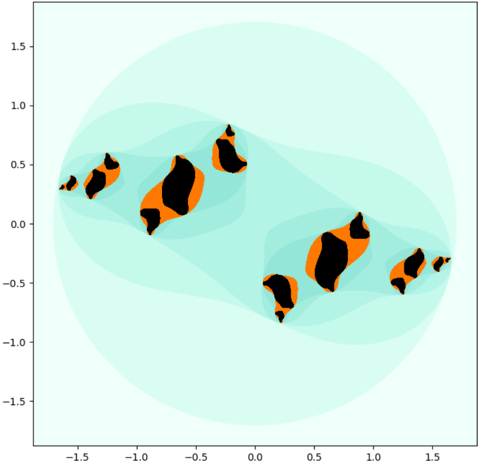

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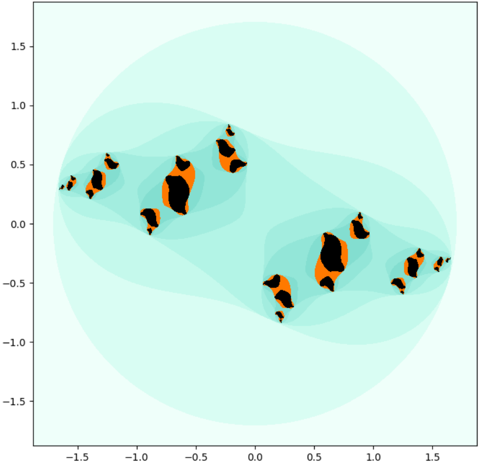

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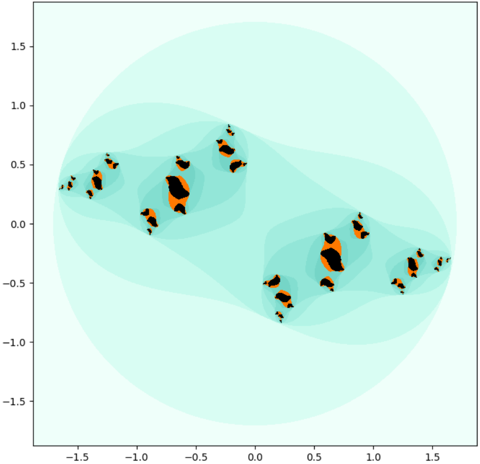

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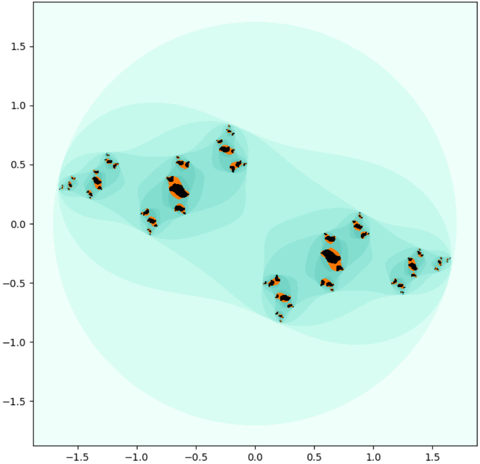

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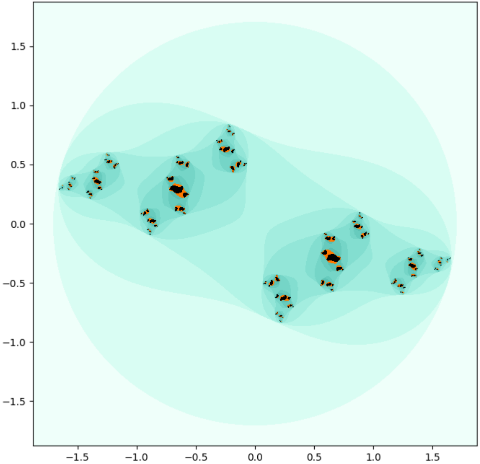

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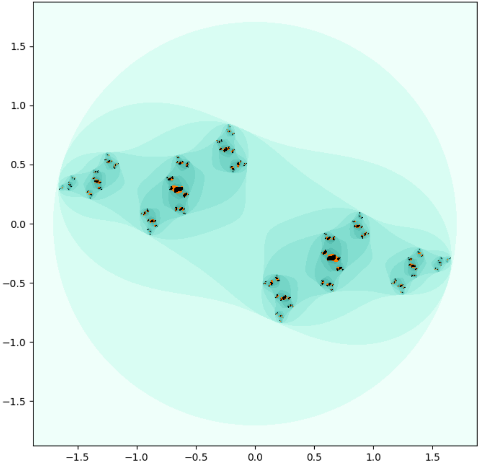

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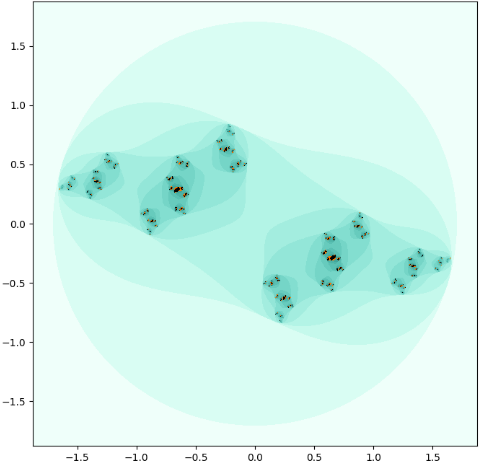

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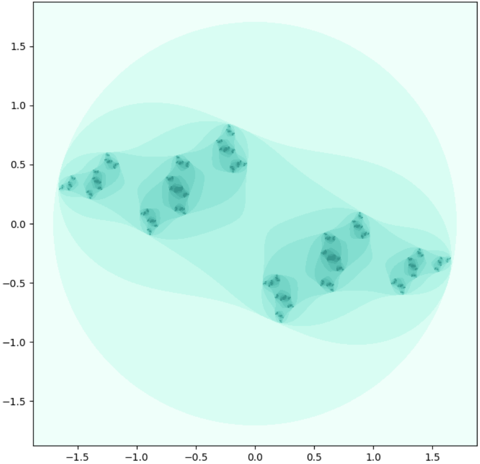

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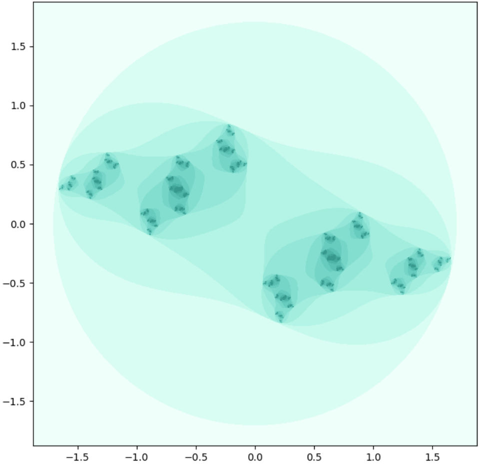
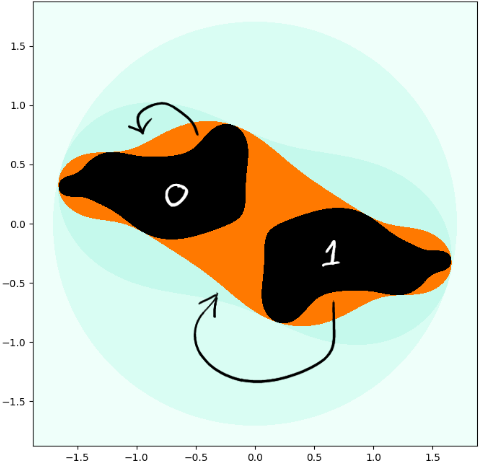


$$c = -1 + \frac{2}{3}i$$

- In the end, the filled Julia set breaks down into infinitely many separate points.
- This kind of Julia set is called *Cantor dust*.

Special kinds of filled Julia sets



$$c = -1 + \frac{2}{3}i$$

- After the first split, we get two "1st-level pieces."
- Call them I_0 and I_1.

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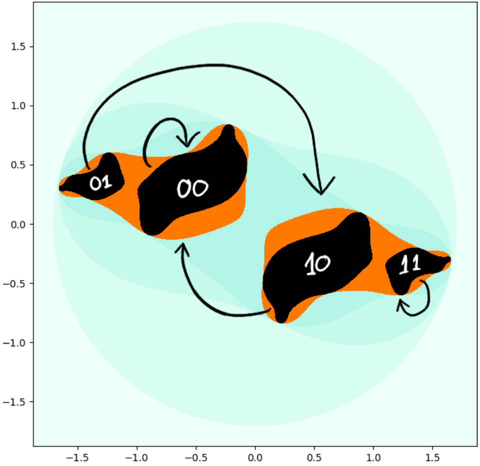

$$c = -1 + \frac{2}{3}i$$

- After the first split, we get two "1st-level pieces."
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- Label the each 2nd-level piece according to where it's sent by Q_c, like we did before.

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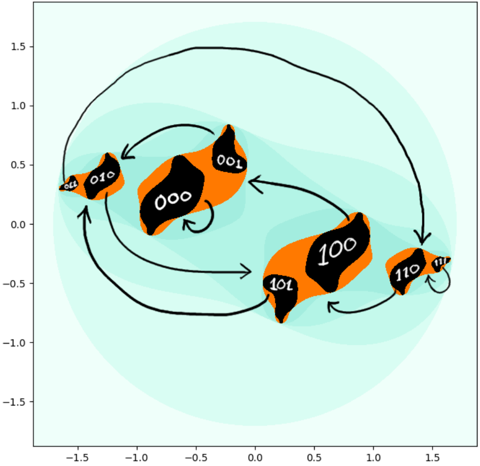

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- After the first split, we get two "1st-level pieces."
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- Label the each 2nd-level piece according to where it's sent by Q_c, like we did before.
- Same for each 3rd-level piece.

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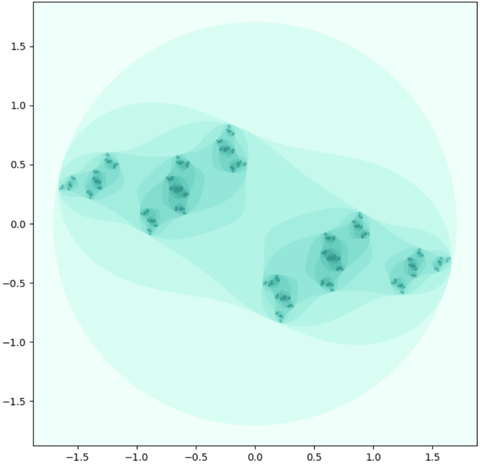


$$c = -1 + \frac{2}{3}i$$

- We can define an itinerary map

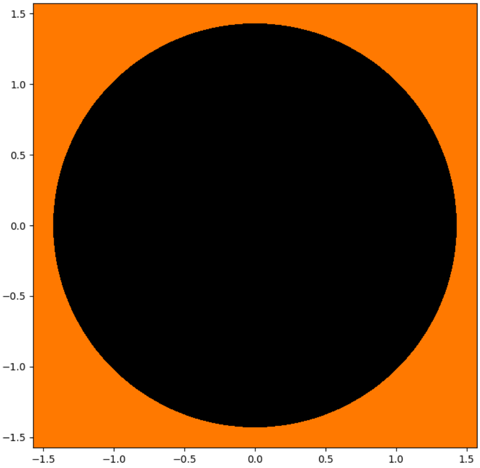
$$\tau \colon K_c \to \mathbf{2}^{\mathbb{N}}.$$

- It's a conjugacy from

$$Q_c \colon K_c \to K_c.$$

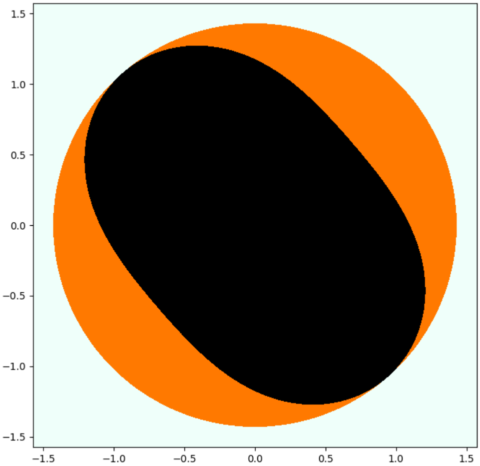
  to the shift map.
- This works whenever K_c is Cantor dust.

Special kinds of filled Julia sets



$$c = \frac{e^{i\theta}}{2}\left(1 - \frac{e^{i\theta}}{2}\right)$$
$$\theta = 2\pi\left(\sqrt{5/3} - 1\right)$$

Special kinds of filled Julia sets



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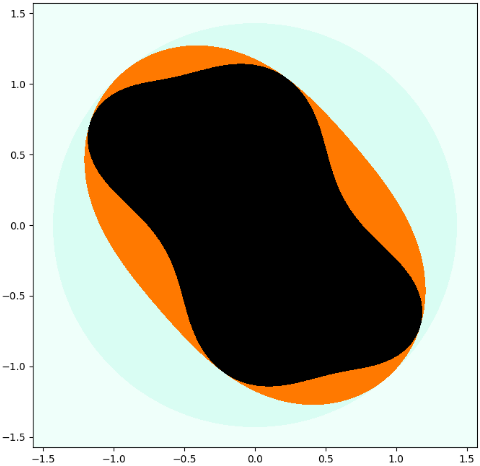

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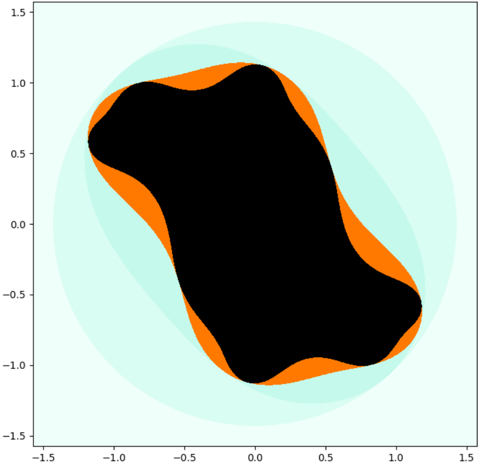

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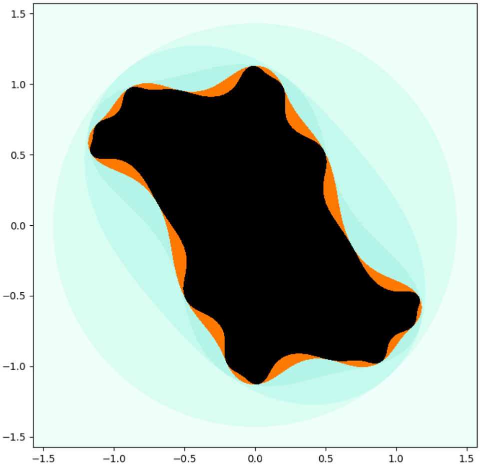

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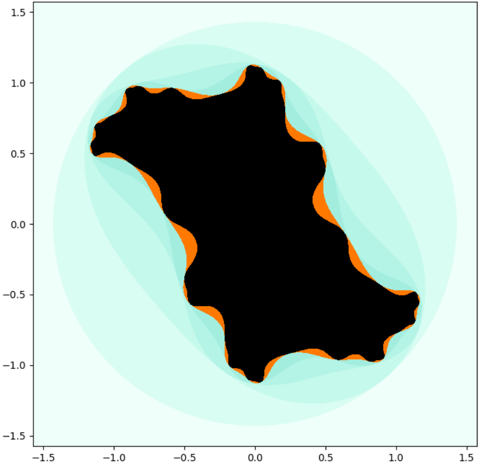

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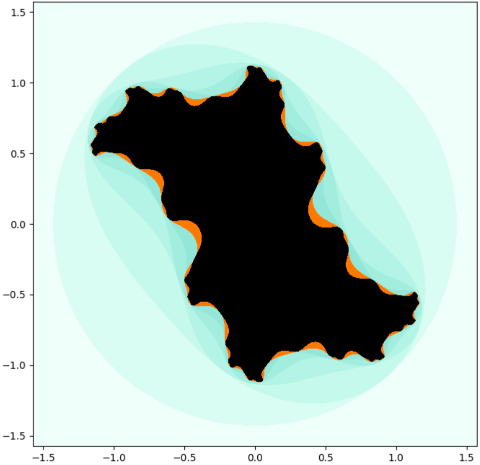

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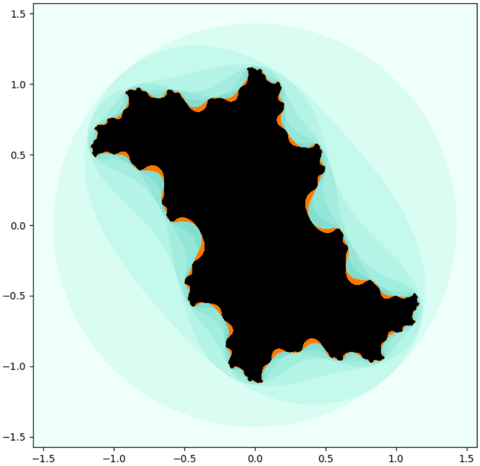

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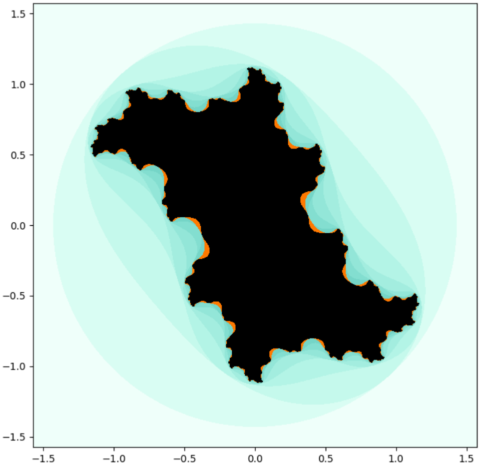

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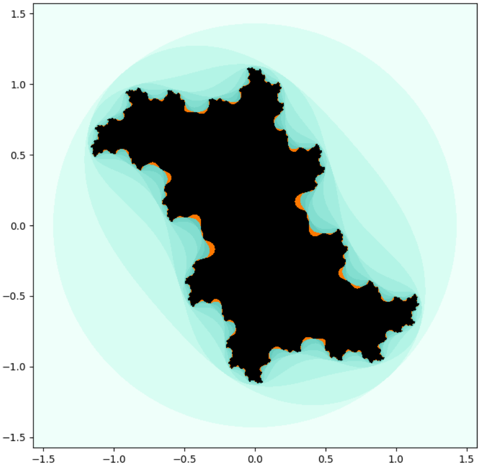

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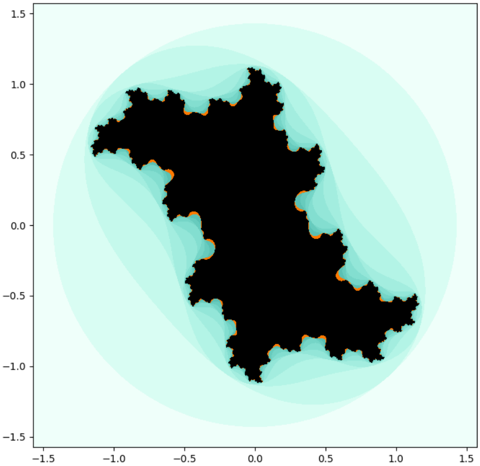

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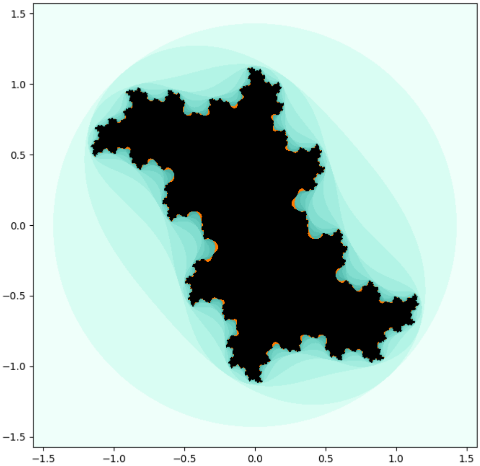

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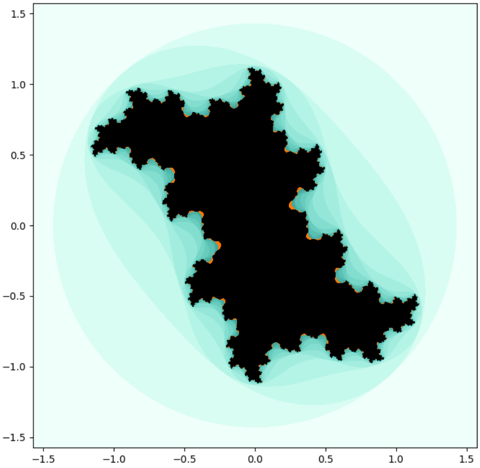

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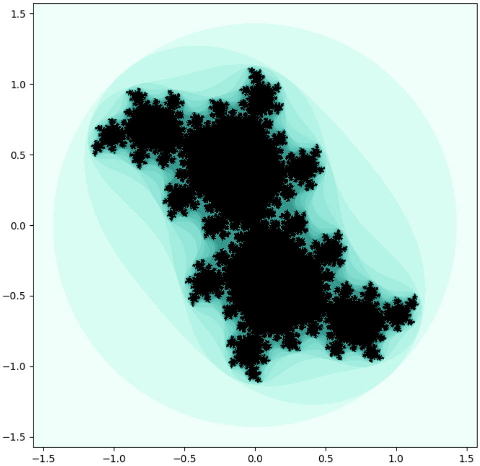

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- This Julia set doesn't quite break into separate pieces.
- Neighboring lobes touch at a single point.

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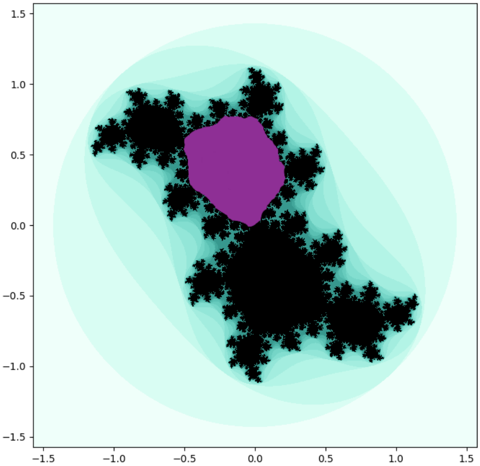

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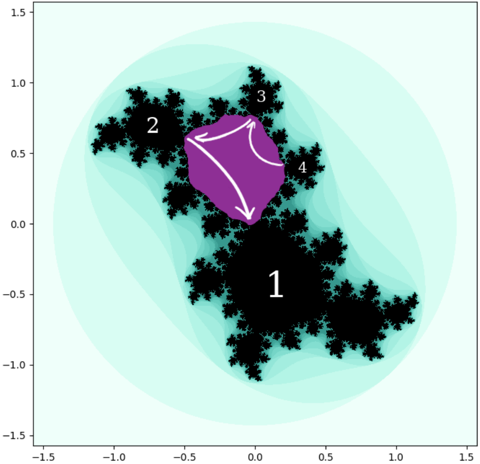
- Neighboring lobes touch at a single point.
- One lobe maps to itself. It's called the *Siegel disk*.

Special kinds of filled Julia sets



$$c = \frac{e^{i\theta}}{2}\left(1 - \frac{e^{i\theta}}{2}\right)$$
$$\theta = 2\pi(\sqrt{5/3} - 1)$$

- On the boundary of the Siegel disk, Q_c is conjugate to R_θ.
- The lobe labeled n maps to the Siegel disk after n steps.

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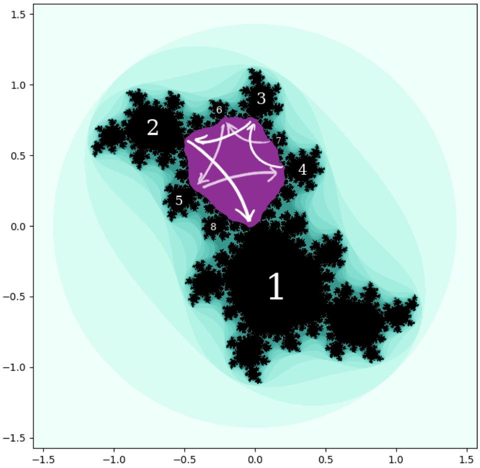

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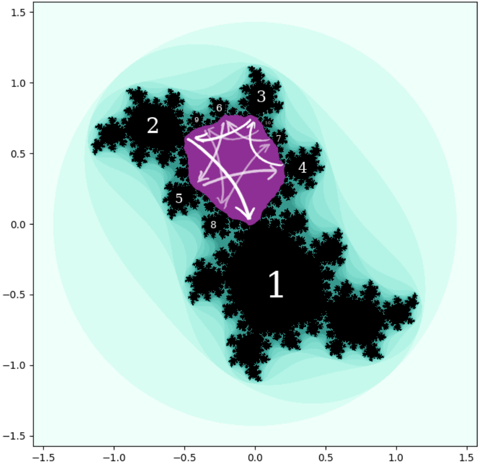

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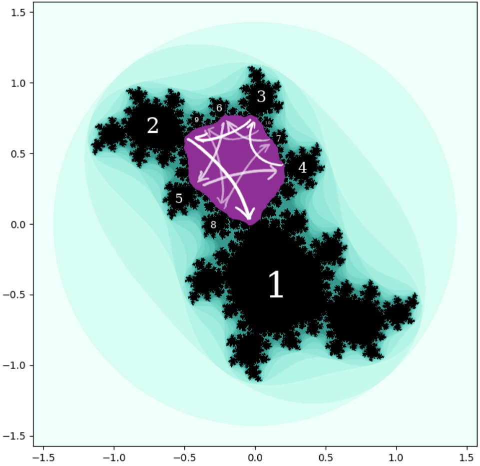

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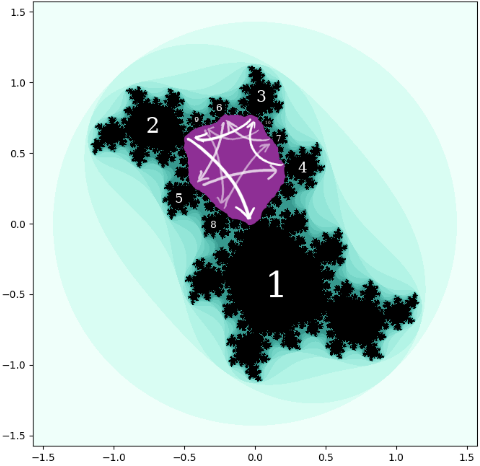

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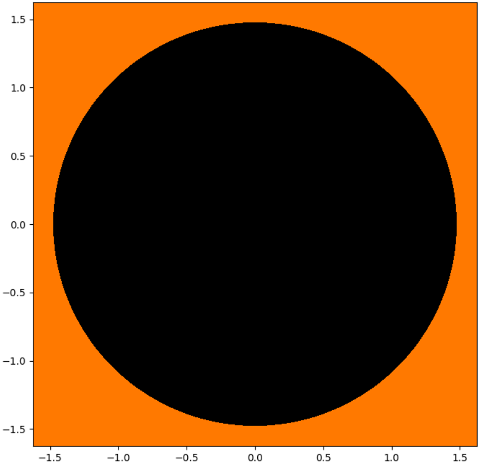

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- The formula for c shown above makes K_c a Siegel cactus whenever $\frac{\theta}{2\pi}$ is irrational.

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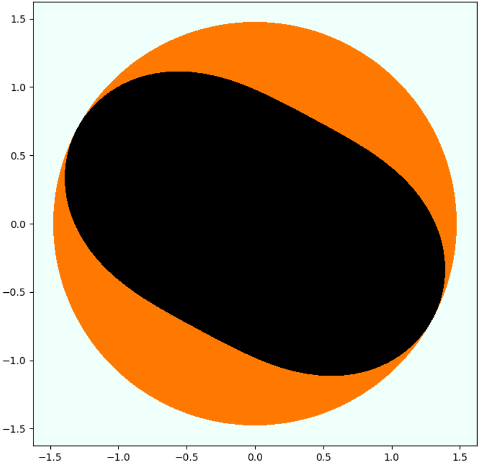

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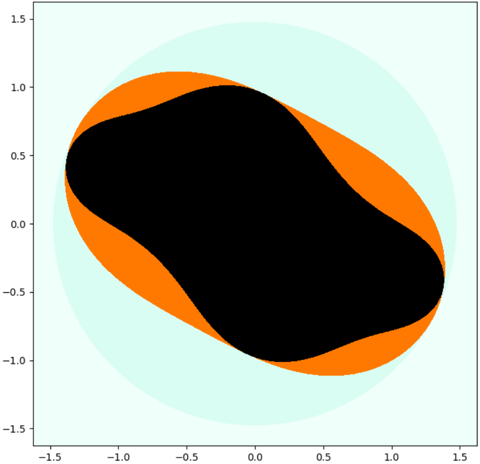

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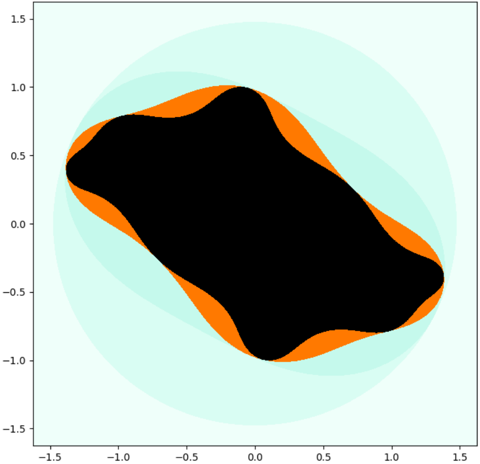

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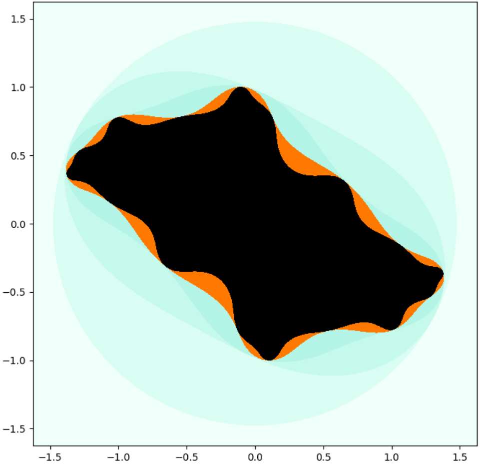

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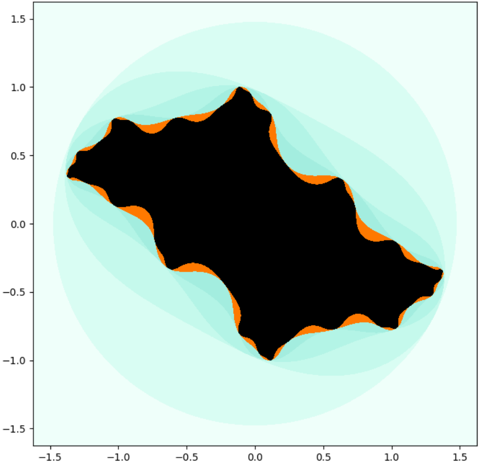

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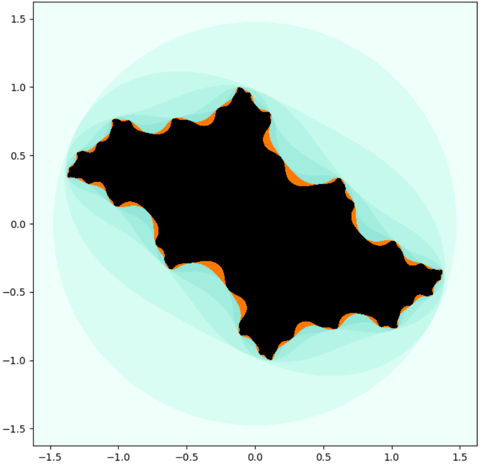

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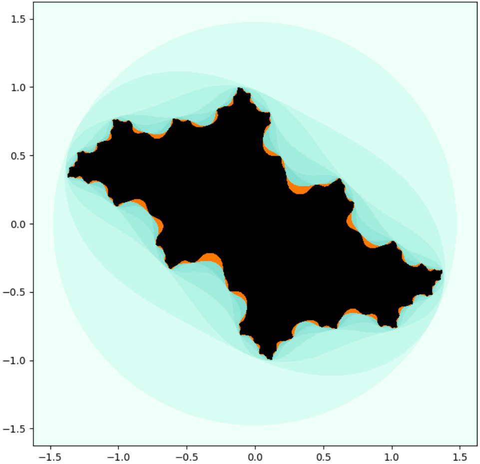

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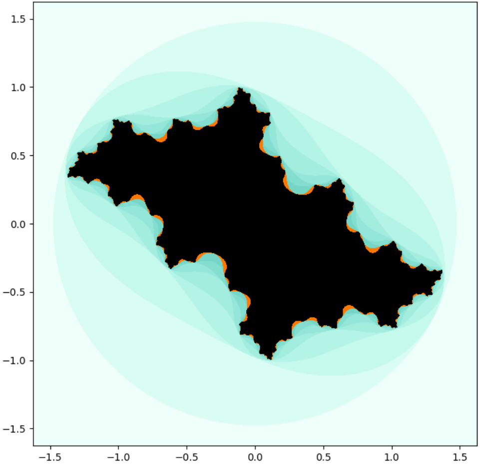

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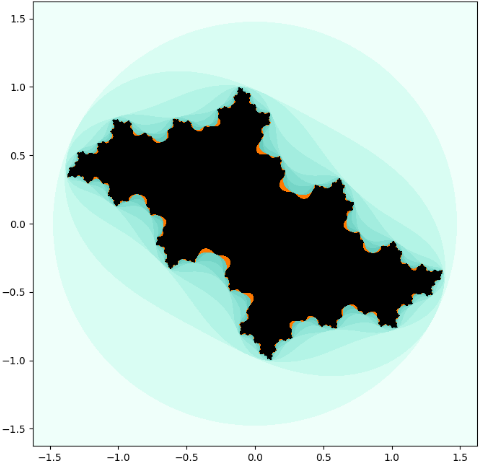

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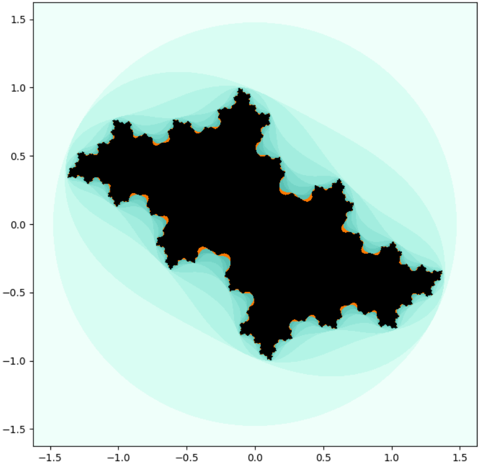

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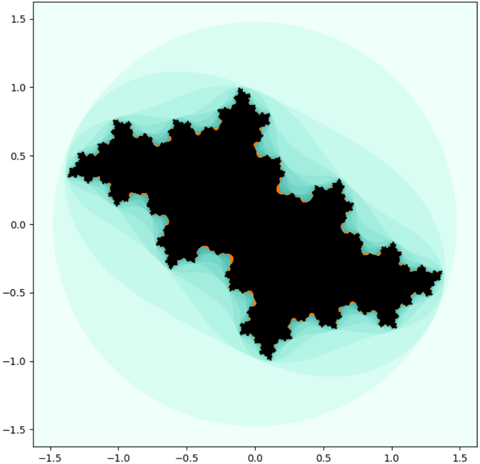

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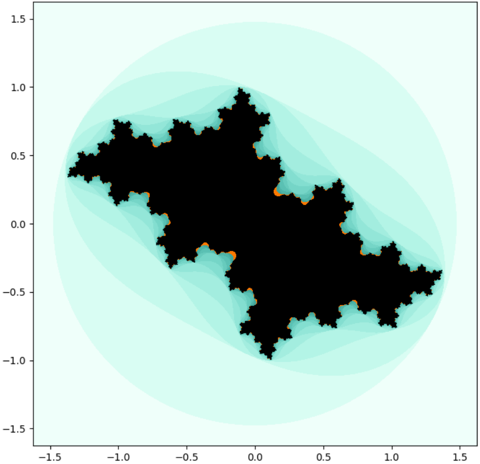

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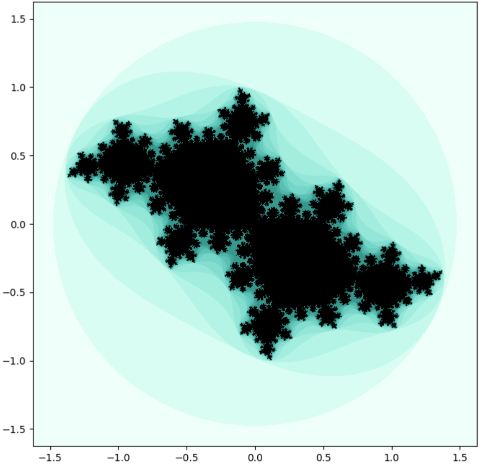

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