Fact sheet

For term test 2 March 12 Chaos, fractals, and dynamics MAT 335, Winter 2019

## Attraction and repulsion

Say p is a fixed point of a dynamical map F.

- A basin of attraction for p is an open ball U with the following properties.
  - $\diamond U$  contains p.
  - $\diamond$  Every orbit starting in U stays in U forever.
  - $\diamond$  Every orbit starting in U limits to p.

(The second property is equivalent to the property that  $F(U) \subset U$ , using our shorthand from class.)

- A region of repulsion for p is an open ball U with the following properties.
  - $\diamond U$  contains p.
  - $\diamond$  Every orbit starting in U eventually leaves U, unless it starts at p. (It only has to leave once; it can come back later.)

## Continuity

**Formal definition.** Consider a function  $\psi: W \to X$ . Pick any point  $w \in W$ . The function  $\psi$  is *continuous at* w if we can keep the output of  $\psi$  within any "target" open ball around  $\psi(w)$  by keeping its input within a small enough open ball around w.

The function  $\psi$  is *continuous* if it's continuous at every point in W.

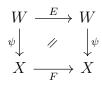
## Semiconjugacy

The function  $\psi: W \to X$  is a *semiconjugacy* from the dynamical map  $E: W \to W$  to the dynamical map  $F: X \to X$  if it has the following properties.<sup>1</sup>

• We can find out what F does to a point by looking at what E does to its label. In symbols,

$$F(\psi(w)) = \psi(E(w))$$
 for all labels  $w \in W$ .

In a picture,



<sup>&</sup>lt;sup>1</sup>The version handed out during the test had a type: E and F were switched. I announced a correction.

- Every point in X has a label. In other words, every point  $x \in X$  can be expressed as  $\psi(w)$  for some  $w \in W$ . A function  $\psi$  with this property is called *onto*.
- Each point in X has a limited number of labels. Specifically, we can fix a maximum m and say that that each point in X has at most m labels in W. A function  $\psi$  with this property is called *at most m-to-one*.
- The function  $\psi$  is continuous.

**Fact.** If  $\psi$  is a semiconjugacy from  $E: W \to W$  to  $F: X \to X$ , it's also a semiconjugacy from  $E^n$  to  $F^n$ , for any number n of iterations.

## The binary representation

The binary representation of angles is a function  $\phi: \mathbf{2}^{\mathbb{N}} \to \mathbb{T}$ . Using  $w_1, w_2, w_3, w_4, \ldots$  to denote the digits of a point  $w \in \mathbf{2}^{\mathbb{N}}$ , we can write

$$\phi(w) \equiv 2\pi \left(\frac{w_1}{2^1} + \frac{w_2}{2^2} + \frac{w_3}{2^3} + \frac{w_4}{2^4} + \ldots\right).$$

**Fact.** The binary representation is a semiconjugacy from the shift map  $S: \mathbf{2}^{\mathbb{N}} \to \mathbf{2}^{\mathbb{N}}$  to the doubling map  $D: \mathbb{T} \to \mathbb{T}$ .

**Fact.** For any  $t \in (-1, 1)$ ,

$$1 + t + t^{2} + t^{3} + t^{4} + \ldots = \frac{1}{1 - t}.$$