Homework 1

Due on Crowdmark
January 17, 11 a.m.

Chaos, fractals, and dynamics
MAT 335, Winter 2019

Show your calculations, and explain your reasoning. Your goal is for the graders to understand how you got your answers, and to be convinced that your reasoning makes sense.

1 Friendly formulas for iterates

Consider the map $F(x) = 2x + 1$ on the state space $\mathbb{R}$.

a. Write formulas for $F^2(x)$, $F^3(x)$, and $F^4(x)$ in terms of x.

b. Write a formula for $F^n(x)$ in terms of n and x.

2 Periodic points of the shift map

For the shift map $S\colon \mathbf{2}^{\mathbb{N}} \to \mathbf{2}^{\mathbb{N}}$, list the periodic points...

a. ... with minimum period 1.

b. ... with minimum period 2.

c. ... with minimum period 3.

3 Eventually fixed points of a quadratic map

Consider the dynamical system with state space $\mathbb{R}$ and dynamical map $G(x) = x^2 - 2$.

a. Find the fixed points of G.

b. Find every point whose orbit reaches a fixed point within four steps.

c. Draw a "family tree" for the points you found in part b, with an arrow from each point y to its "parent" $G(y)$. Don't forget to draw an arrow from each fixed point to itself!

4 Sweep away the `1`s

Here's a new dynamical system.

State space: $\mathbf{2}^{\mathbb{N}}$.

Dynamical map: Each `1` that's followed by a `0` turns into a `0`.

Let's call this map A. As a demonstration, here's what A does to one point in $\mathbf{2}^{\mathbb{N}}$.

$$\begin{aligned} w &= \texttt{001110011011110100101110}\ldots \\ A(w) &= \texttt{0011}\underline{\texttt{0}}\texttt{001}\underline{\texttt{0}}\texttt{0111}\underline{\texttt{0}}\texttt{0}\underline{\texttt{0}}\texttt{00}\underline{\texttt{0}}\texttt{011}\underline{\texttt{0}}\texttt{0}\ldots \end{aligned}$$

The changed digits are underlined.

a. Describe all the fixed points of A.

b. Find a point in $\mathbf{2}^{\mathbb{N}}$ which is not eventually fixed. Describe your point carefully, and convince a skeptical grader that it's not eventually fixed.

5 Dueling periods

Consider a dynamical system with state space Y and dynamical map M.

a. Suppose $a \in Y$ is both 2-periodic and 3-periodic. Find the minimum period of a.

b. Suppose $b \in Y$ is both 3-periodic and 5-periodic. Find the minimum period of b.