MAT244 - Tutorial 01 - C.J. Adkins

Chapter 1 Material

What is an O.D.E? $F(\underbrace{t, u(t)}_{\text{solution}}, \underbrace{u'(t), \ldots, u^{(n)}(t)}_{\text{it's derivatives}}) = 0$

What makes them different? Order = Largest derivative $(n \in \mathbb{N})$, Basically $F(\cdot)$

Ex (order) (1.3 - # 6) What is the order?

$$y''' + t y' + (\cos^2 t) y = t^3 \implies \text{order} = 3$$

$$3 \qquad 1 \qquad 0$$

Also, in this case $F(t, y, y', y'', y''') = y''' + t y' + (\cos^2 t) y - t^3$

Ex (Solutions) (1.3 - # 12) Check the solution.

$$t^2 y'' + 5t y' + 4y = 0, \quad t > 0 \qquad y_1(t) = \frac{1}{t^2}, \quad y_2(t) = \frac{\ln(t)}{t^2}$$

Remark (linear equations) - If y_1 & y_2 are solutions, then $y_1 + y_2$ is a solution.

Check $y = y_1 + y_2 = \dfrac{1 + \ln(t)}{t^2}$, compute $y' = -\dfrac{1 + 2\ln(t)}{t^3}$.

compute $y'' = \dfrac{1 + 6\ln(t)}{t^4}$, Now lets check R.H.S = L.H.S?

$$0 \overset{?}{=} t^2 \frac{(1 + 6\ln(t))}{t^4} - 5t \frac{(1 + 2\ln(t))}{t^3} + 4 \frac{(1 + \ln(t))}{t^2} = 0 \checkmark$$

Therefore $y(t)$ is a solution.

Ex (Solutions) (1.3 - # 14) Check solution.

$$y' - 2ty = 1, \quad y = e^{t^2}\left(1 + \int_0^t e^{-s^2} ds\right)$$

$$y' = 2te^{t^2}\left(1 + \int_0^t e^{-s^2} ds\right) + \underbrace{e^{t^2}\left(e^{-t^2}\right)}_{\text{FTC}} = 2te^{t^2}\left(1 + \int_0^t e^{-s^2} ds\right) + 1 = 2ty + 1$$

L.H.S = R.H.S $\checkmark$

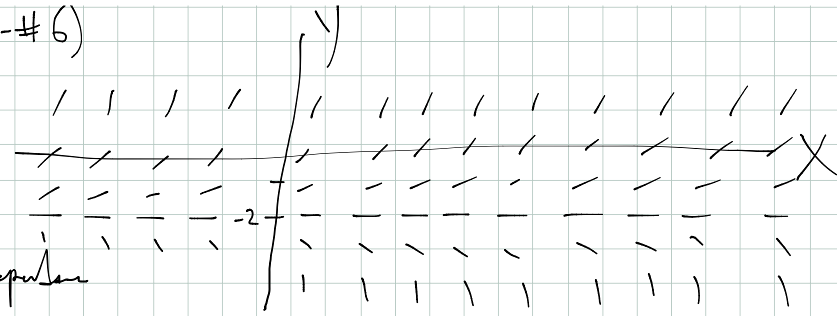
Direction Fields = i.e Slope fields = Possible solutions.

To do this, just calculate $y'(x, y)$. Draw $\mathbb{R}^2$

Ex (1.1-#6)

$y' = y+2$

Notice no dependence on x!



Big points, $y=-2 \Rightarrow y'=0$ (flat), if $y>-2$ then $y'>0$
if $y<-2$ then $y'<0$

What does $y(0)=C$ mean here? Picks a solution! Growth at ∞?
if $y(0)=-2 \Longrightarrow y=-2 \ \forall x \in \mathbb{R}$, if $y(0)>-2 \Rightarrow y \xrightarrow[x\to\infty]{} \infty$
if $y(0)<-2 \Rightarrow y \xrightarrow[x\to\infty]{} -\infty$

Math lingo - Integral Curves are what we call the solutions to the O.D.E

Section 1.2 gives examples of how to solve O.D.E.

Ex (1.2 #15) Solve Newton's Law of Cooling

$$\frac{du}{dt} = -K(u-T), \quad u(t) = \text{Temperature}, \ T \text{ is ambient temp}, \ K>0, \ u(0)=u_0$$

a) solve for $u(t)$, rewrite Eq. to an integral Eq

$$\int \frac{du}{u-T} = \int -K\,dt \Longleftrightarrow \ln(u-T) = -Kt + C$$

$$\Longleftrightarrow u(t) = T + \exp(-Kt + C)$$

$$= T + \underbrace{\tilde{C}}_{\text{new const} = e^C} \exp(-Kt)$$

What about Initial conditions?

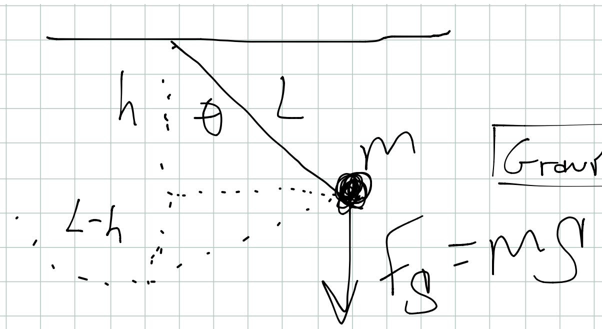
$$u(0) = T + \tilde{C} = u_0 \Longleftrightarrow \tilde{C} = u_0 - T$$

$$\therefore u(t) = T + (u_0 - T)e^{-Kt}$$

b) find τ when $\dfrac{u(\tau)-T}{u(0)-T} = \frac{1}{2} \Longleftrightarrow e^{-K\tau} = \frac{1}{2} \Longleftrightarrow K\tau = \ln(2)$

$$\Longleftrightarrow \tau = \frac{\ln(2)}{K}$$

Ex (1.3 #30) We'll derive the pendulum equation by conservation of Energy.



$\boxed{\text{Gravity}}$

Kinetic Energy $= \frac{1}{2}mv^2 = \frac{1}{2}m(L\omega)^2$ where $\omega = \frac{d\theta}{dt}$ is the angular velocity

$$= \frac{1}{2}mL^2\left(\frac{d\theta}{dt}\right)^2$$

Potential Energy $= mg\underbrace{(L-h)}_{\text{height of mass}} = mg(L - L\cos\theta) = mgL(1-\cos\theta)$

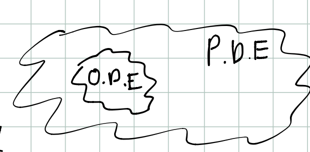
Energy $= KE + PE = mL\left(\frac{1}{2}\left(\frac{d\theta}{dt}\right)^2 L + g(1-\cos\theta)\right) = \underline{\text{Const}}$

$$\therefore \frac{dE}{dt} = 0 = mL\left(\frac{d\theta}{dt}\frac{d^2\theta}{dt^2}L + g\sin\theta\frac{d\theta}{dt}\right)$$ Conservation of Energy

$$= mL\frac{d\theta}{dt}\left(\frac{d^2\theta}{dt^2}L + g\sin\theta\right)$$

$$= 0 \iff \frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$

A few words on P.D.E



- More complicated, O.D.E is a subset.
- solution are in multivariables
- Notation and definitions come from P.D.E

Quiz & Questions

Next week (Methods for 1st order O.D.E)