Tutorial #9

MAT 188 – Linear Algebra I – Fall 2015

SOLUTIONS

Problems (Please note these are from G. Strang's and W.K. Nicholson's Linear Algebra Texts)

Question 1 Explain why the line that passes through the points (x_1, y_1), and (x_2, y_2) can be expressed as

$$\det \begin{pmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{pmatrix}$$

and use this approach to find the equation of the line that passes through the points $(2, 8)$ and $(4, 7)$.

Solution We know the determinate of a matrix is zero if two rows are linearly dependent. The top row is dependent if it is the line passes through both points. If we expanded the determinant we see

$$\det A = x(y_1 - y_2) - y(x_1 - x_2) + (x_1 y_2 - x_2 y_1) = 0 \implies y = x\frac{y_1 - y_2}{x_1 - x_2} + \frac{x_1 y_2 - x_2 y_1}{x_1 - x_2}$$

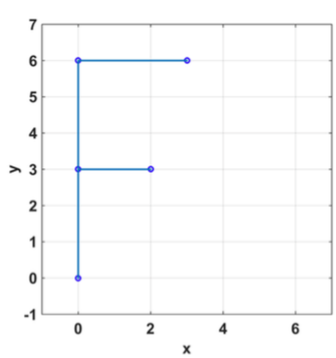
Using the numbers given we obtain

$$\boxed{y = -\frac{x}{2} + 9}$$

□

Question 2 In computer graphics, a simple way of representing an arbitrary curve is to approximate it as a set of line segments. Let us assume, for example, that the letter "F" is represented as depicted in the figure, using 5 vertices, which are the columns of matrix F:

$$F = \begin{pmatrix} 0 & 0 & 2 & 0 & 3 \\ 0 & 3 & 3 & 6 & 6 \end{pmatrix}$$



Consider the transformation $T(x) = Ax$, where A is a 2×2 matrix applied to the letter "F". Use $\det(A)$ to indicate if the letter is shrunk, expanded, or reflected for

$$a) A = \begin{pmatrix} 0.8 & 0 \\ 0 & 1 \end{pmatrix}, \quad b) A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad c) A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \quad d) A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Solution

$$a) \det A = 0.8 \implies \text{Shrunk}$$

$$b) \det A = -1 \implies \text{Reflected}$$

$$c) \det A = 4 \implies \text{Expanded}$$

$$d) \det A = 1 \implies \text{Rotated}$$

□

Question 3 Determine the volume of the parallelepiped with sides described by the vectors $u = (3, 5, 2)^T$, $v = (6, 1, 3)^T$, and $w = (2, 0, 4)^T$.

Solution If we transformed the unit cube by $T(x) = Ax$, where

$$A = \begin{pmatrix} 3 & 6 & 2 \\ 5 & 1 & 0 \\ 2 & 3 & 4 \end{pmatrix}$$

We'll have T(cube)=parallelepiped. Thus since determinate tell us how volumes change, we have

$$\text{Volume of parallelepiped } = |\det A| \times \text{Volume of Cube}$$

Since the unit cube has unit volume, we compute the determinate.

$$\det A = 3(1 \times 4) - 6(5 \times 4) + 2(5 \times 3 - 2 \times 1) = 12 - 120 + 26 = -82$$

Therefore

$$\boxed{\text{Volume of parallelepiped } = 82 \text{ units}^3}$$

□