Tutorial #6

MAT 188 – Linear Algebra I – Fall 2015

SOLUTIONS

Problems (Please note these are from Holt's Linear Algebra Text and D. Poole's Text)

3.2 - #29 Holt Suppose that A is a 3×3 matrix. Find a 3×3 matrix E such that the product EA is equal to A with:

(a) The first and second rows interchanged.

(b) The first and third rows interchanged.

(c) The second row multiplied by -2.

Solution We know the identity matrix produces $IA = A$, thus we simply change the rows and values of the identity. Specifically

$$\boxed{a) \quad E = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad b) \quad E = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad c) \quad E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

3.2 - # 26 Poole Find conditions on a, b, c, d such that

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

commutes with both

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \& \quad C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

Solution Remember that two matrices commute if $AB = BA$. Let's check:

$$AB = BA \implies \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \implies \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \implies \begin{cases} a = a, & b = 0 \\ c = 0, & 0 = 0 \end{cases}$$

$$AC = CA \implies \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \implies \begin{pmatrix} 0 & b \\ 0 & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ c & d \end{pmatrix} \implies \begin{cases} 0 = 0, & b = 0 \\ c = 0, & d = d \end{cases}$$

Thus we see

$$\boxed{b = c = 0 \quad \& \quad a, b \in \mathbb{R}}$$

3.3 - #19 Holt Solve the following linear system by finding the inverse of the coefficient matrix.

$$\begin{array}{ccccccc} 3x_1 & - & x_2 & + & 9x_3 & = & 4 \\ x_1 & - & x_2 & + & 4x_3 & = & -1 \\ 2x_1 & - & 2x_2 & + & 10x_3 & = & 3 \end{array}$$

Solution It's easy to see the coefficient matrix is given by

$$A = \begin{pmatrix} 3 & -1 & 9 \\ 1 & -1 & 4 \\ 2 & -2 & 10 \end{pmatrix} \implies Ax = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

We know the inverse may be found by putting the following matrix into reduced row echelon form:

$$\left(\begin{array}{ccc|ccc} 3 & -1 & 9 & 1 & 0 & 0 \\ 1 & -1 & 4 & 0 & 1 & 0 \\ 2 & -2 & 10 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -1 & 4 & 0 & 1 & 0 \\ 3 & -1 & 9 & 1 & 0 & 0 \\ 2 & -2 & 10 & 0 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 4 & 0 & 1 & 0 \\ 0 & 2 & -3 & 1 & -3 & 0 \\ 0 & 0 & 2 & 0 & -2 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 4 & 0 & 1 & 0 \\ 0 & 1 & -3/2 & 1/2 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1/2 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 2 & -5/4 \\ 0 & 1 & 0 & 1/2 & -3 & 3/4 \\ 0 & 0 & 1 & 0 & -1 & 1/2 \end{array}\right)$$

Thus the inverse matrix is given by

$$A^{-1} = \begin{pmatrix} 1/2 & 2 & -5/4 \\ 1/2 & -3 & 3/4 \\ 0 & -1 & 1/2 \end{pmatrix}$$

Therefore we conclude

$$\boxed{x = A^{-1}b = \begin{pmatrix} 1/2 & 2 & -5/4 \\ 1/2 & -3 & 3/4 \\ 0 & -1 & 1/2 \end{pmatrix} \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -15 \\ 29 \\ 10 \end{pmatrix}}$$